A method for noise removal and object detection based on data expansion by multiresolution representation and data compression by principal component analysis

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Abstract

This paper presents a method for noisy object detection which is based on the expansion/compression paradigm and combines a multiresolution approach with the principal component analysis (PCA). The multiresolution representation is done by successive Gaussian filterings. The compression of the expanded information is achieved by only keeping the first PCA factorial image. Finally, the object of interest is detected and delineated from the previous factorial image by using a standard valley thresholding technique. The proposed method behaves as a compromise between the various Gaussian filterings by limiting the blurring effect of such filterings and removing most of the noise. The experimental evaluation using synthetic objects has shown the ability of this approach to clean strongly noisy images. For scenes containing several objects of interest, like CT-Scan images, we first search for regions of interest (ROIs). Then, for each ROI, we locally apply the proposed detection method. Experimental results have shown the potential of the proposed method for the detection of liver tumours from CT-Scan images and for the segmentation of handwritten characters.

1. Introduction

The detection of noisy objects is a classical problem encountered in most vision systems. To solve this problem, many approaches and techniques have been developed. A classical approach is the region based approach. The goal of methods based on such an approach is to smooth the different regions without removing the boundaries between them. Image smoothing can be performed using either isotropic diffusion or anisotropic diffusion. The current limitation of isotropic diffusion is the blurring effect of edges and the unreliable correct noise removal when no knowledge about the noise is available [6]. To avoid these limitations, Perona and Malik [8] have proposed the anisotropic diffusion which consists in smoothing the regions without blurring their edges. But the quality of this smoothing strongly depends on the number of iterations, which may lead to a really time consuming processing [5]. Moreover, this approach requires a supervised definition of several parameters. To overcome these drawbacks, we propose an unsupervised method based on the expansion/compression paradigm [7] which combines a multiresolution approach [1] and the principal component analysis (PCA) [9]. The multiresolution representation is achieved by successive Gaussian filterings. The compression of the expanded information is carried out using PCA. Then, the denoised image is the so called first factorial image obtained by keeping the highest eigenvalue only. This unsupervised method aims to find the best compromise between a too strong blurring and a too weak noise removal.

In the following section, we describe the detection method. In the third section, we study the behaviour of the proposed method with respect to the degree of noise and the object size. In the fourth section, we illustrate the potential of the proposed method for liver tumour detection and segmentation of handwritten characters. Finally, we conclude on this unsupervised detection method.

2. Method

The proposed detection method is based on the expansion/compression paradigm (E/C)[7]. This is a general approach for extracting features for pattern recognition applications. Essentially, this E/C paradigm first expands the input signal in some transform domain and then compresses the resulting expansion for presenting to a classifier. In our case, the transformation is the Gaussian transform and consists in applying a succession of Gaussian filterings of in-
creasing width to the original noisy image \( f(x, y) \). Each Gaussian filtering, characterized by the width of the Gaussian filter \( \sigma_i \), leads to a transformed image \( f_i(x, y) \):

\[
f_i(x, y) = \int \int f(a, b) \exp\left[-\frac{(a-x)^2 + (b-y)^2}{2\sigma_i^2}\right] \, da \, db.
\]

Such Gaussian filtering leads to the multiresolution representation [1, 2] of the original noisy image \( f(x, y) \). As usually considered in multiresolution analysis, we operate an octave decomposition by choosing \( \sigma_i = 2^i \), with \( i = [0, 1, ..., n] \). Then, the compression enables to reduce the redundancy of the representation and consists in the principal component analysis (PCA). To perform such a compression, we consider each Gaussian filtered image \( f_i(x, y) \) as a column vector \( \bar{v}_i \) whose length \( k \) is equal to the number of pixels of the analyzed image. Then, the PCA is applied to the matrix \( X \) whose columns are the \( n \) image vectors \( \bar{v}_i \):

\[
X = [\bar{v}_1, ..., \bar{v}_n].
\]

The PCA decomposes the information contained in image data by creating a series of so-called factorial images \( F_1, F_2, ... \) of decreasing variance. To begin this transformation, image vectors \( \bar{v}_i \) are centered by subtraction of their mean \( \mu_i \) and normalized by their standard deviation \( s_i \). This leads to the data matrix \( A \):

\[
A = [\bar{t}_1, ..., \bar{t}_n].
\]

Then, the correlation matrix, \( C \), is computed:

\[
C = \frac{1}{K} A^T A,
\]

where the superscript \( T \) represents the matrix transpose operator.

The PCA states that there is an orthogonal matrix \( V = \begin{bmatrix} \bar{v}_1 & \bar{v}_2 & \cdots & \bar{v}_n \end{bmatrix} \) and a diagonal matrix \( D \) such that \( A^T A = V D V^T \). The columns of \( V \) are the eigenvectors of \( A^T A \) and form an orthonormal basis. The diagonal entries of \( D \) are the eigenvalues \( \lambda_i \) of \( A^T A \) and are sorted by decreasing order so that \( \lambda_j \geq \lambda_{j+1} \) for \( j = 1, ..., n \).

The first \( p \) factorial images \( \bar{F}_i \), with \( i = 1, 2, ..., p \), which correspond to eigenvalues \( \lambda_i \), are computed by projection of centered and normalized image data on the eigenvectors \( \bar{v}_1, ..., \bar{v}_p \):

\[
\bar{F}_i = A \times \bar{v}_i.
\]

\( \bar{F}_i \) denotes the factorial image vector related to the factorial image \( F_i \). These linear projections lead to \( p \) uncorrelated factorial images. The choice of the number \( p \) of factorial images will be based on a visual perception criteria as explained hereafter.

The figure 1 gives the example of a noisy disc detected by the proposed method. The first line represents the initial noisy disc and the set of six Gaussian filterings used for the expansion. The second line represents the set of factorial images, sorted by decreasing order of their eigenvalues, and the thresholded first factorial image (left side). The visual observation of the first factorial image clearly shows that most of the noise has been removed without blurring the object boundary. Furthermore, this factorial image explains about 90% of the total variance of the set of centered and normalized Gaussian filtered images. The following factorial images \( (F_2, F_3, ...) \) are more noisy and do not contain useful additional information from the point of view of visual perception. Consequently, it is enough to only consider the first factorial image \( F_1 \).

Finally, the object is detected and delineated using the standard valley thresholding technique which is applied to the first factorial image \( F_1 \). This thresholding is illustrated by the binary object on the left side of the second line of the figure 1. This shows that the noisy disc has been correctly delineated. This furthermore justifies the choice of only keeping the first factorial image.

The first factorial image is a linear combination of the six Gaussian filterings which can be interpreted as a compromise between a too weak smoothing which would not remove enough noise and a too strong filtering which would blur the object boundaries.

### 3. Evaluation of the object detection method

We propose to study the behaviour of the proposed method with respect to the type and degree of noise corrupting the images and to the object size. For this analysis, we consider a given set of Gaussian filters defined by \( \sigma_i = [1, 2, 4, 8, 16, 32] \).
3.1. Influence of Gaussian additive noise

We assume that the signal $f(x, y)$ we want to denoise is an ideal signal corrupted by an additive noise. We propose to represent the noise as a normally distributed (Gaussian), zero-mean random process with a probability density function $f_{x}(x)$ [3]:

$$f_{x}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$  \hspace{1cm} \text{(6)}

where $\sigma$ is the noise standard deviation. The effect of an additive noise process, $n$, on the image $f(x, y)$ can be defined as the summation of the true signal $s(x, y)$ with the noise:

$$f(x, y) = s(x, y) + n(x, y).$$  \hspace{1cm} \text{(7)}

For this study, we considered that the ideal signal is a disc a diameter 32 pixels corrupted by various additive Gaussian noises leading to signal to noise ratios (SNRs) of $[0.00, 0.704, 12.04, 15.91, 19.08, 27.95]$ in dB. The first line of figure 2 shows the set of noisy discs corresponding to the various SNRs. The second line (resp. third line) shows the resulting factorial images (resp. the resulting delineated discs). We see that the initially noisy discs are correctly delineated.

The factorial image is a linear combination of the six Gaussian filterings whose weights are the coordinates of the first eigenvector. More precisely, in our case: the first coordinate gives the weight of the first Gaussian filtering defined by a width of 1 pixel, the second coordinate gives the weight of the second Gaussian filtering defined by a width of 2 pixels, and so on, up to the strongest Gaussian filtering defined by a width of 32 pixels. The figure 3 shows that Gaussian filtering weights depend on SNRs. In particular, when the noise increases, the influence of the weak Gaussian filterings decreases. This shows that the PCA compression takes automatically into account the degree of noise corrupting the image without any a priori knowledge about it.

3.2. Influence of Gaussian multiplicative noise

Many acquisition systems lead to data corrupted by a multiplicative noise. For this reason, we propose to evaluate the ability of the proposed method to remove such kind of noise. The multiplicative noise can be defined by a unitary-mean normal random process with a probability density function of [3]:

$$f_{x}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-1)^2}{2\sigma^2}}$$  \hspace{1cm} \text{(8)}

where $\sigma$ is the standard deviation of the noise process. The effect of such a multiplicative noise process, $m$, on the image $f(x, y)$ can be defined as being the multiplication of the true signal $s(x, y)$ with the noise:

$$f(x, y) = m(x, y).s(x, y).$$  \hspace{1cm} \text{(9)}
3.3. Influence of object size

We applied the proposed method to noisy discs of varying diameters \(d\) being, in pixels: \(d = [10, 16, 32, 64, 128]\). Each disc was initially corrupted by a Gaussian additive noise corresponding to a SNR of 0 dB. The figure 6 shows the different initially noisy discs (first line), the factorial images (second line) and the resulting thresholdings (third line) obtained for the different sizes. We see that, in each case, the noisy disc has been correctly detected.

The figure 7 gives the distribution of the weights characterizing the PCA compression with respect to the various object sizes. It can be observed that this weight distribution depends on the object size: when the size of the disc decreases, the weights corresponding to both weakest and strongest filtering decreases. This shows that the PCA analysis selects automatically the weights so that small objects do not disappear because of a too strong smoothing.

The figure 8 gives an example of tumour in the liver. **Left side:** example of slice of abdomen. **Right side:** Segmented Liver. The highlighted and thresholded area contains a tumour. This tumour appears as a dark and compact object which is difficult to automatically detect and correctly delineate.
4. Illustration of the potential of the proposed method

The potential of the proposed method is illustrated by processing some images encountered in two different applications: the detection of liver tumours from CT-Scan images and the segmentation of handwritten characters.

4.1. Liver tumour detection from CT-Scan images

The main purpose of the computer aided diagnostic of the liver is to segment automatically the liver and the different structures such as vessels and tumours from a set of about hundred slices acquired by a CT-Scanner [10]. Each slice is an image of 512x512 pixels. The pixel size is $0.65 \times 0.65 \text{mm}^2$.

Our detection method will be applied to the segmentation and delineation of liver tumours, appearing darker than the parenchyma as shown on the figure 8-Left. The figure 8-Right corresponds to the segmented liver within which a region of interest containing the tumour has been thresholded. The granular aspect of this thresholded image shows the difficulty of the tumour delineation. This justifies the need of a smoothing process. Tumours are of varying sizes and of varying contrasts [10]. We first search for regions of interest (ROIs) containing tumours of various sizes. For this purpose, we initially threshold the original image, perform a morphological alternate sequential filtering to remove the granular noise, and then close the binary filtered image using a larger structuring element. The ROIs are obtained by searching for the connected components of the closed thresholded image. Finally, for each ROI, we locally apply our detection method. Additional details are given in our previously published work dealing with the opto-electronic implementation of the proposed method for tumour detection [4]. The figure 9 illustrates the different steps leading to the detected tumour and to three false alarms. The figure 10 shows the details of the processing of the ROI containing the tumour belonging to the slice displayed on the right side. Our detection method has been applied to fifteen liver slices, using four Gaussian filterings defined by $\sigma = [2, 4, 8, 16]$ [4]. The results have been analyzed by the medical staff of our medical partner IRCAD (Institut de Recherche sur le Cancer de l’Appareil Digestif). Several conclusions can be extracted from this analysis [4]. First, most tumours are correctly depicted. Secondly, the segmentation precision has been visually validated as correct.

4.2. Handwritten character segmentation

Currently, there is a substantial and growing interest in the field of document image processing and understanding
Two practical applications have been considered to illustrate the potential of the proposed method. These applications dealt with the liver tumour detection and the segmentation of handwritten characters. Preliminary experimental results are promising. Nevertheless, further experiments must be carried out to validate the efficiency of the proposed method on large databases.

References


