Looming Stereo with Adaptive Windowing

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Abstract

Looming stereo is a stereo configuration where the camera baseline is perpendicular to the image planes. In conventional stereo with parallel or vergent cameras, depth is extracted using disparity. In looming stereo, however, we can compute depth using what we call the **disparity ratio**. The disparity ratio measures relative feature sizes between the two images rather than the relative feature position. To compensate for the significant change of sizes in the front and back images, we also propose a new method of determining **adaptive window** sizes based on signal-to-noise ratio optimization. Our looming stereo algorithm uses both adaptive window correlation and dynamic programming. Error analysis and experimental results show that our looming stereo algorithm produces accurate reconstruction.

1 Introduction

Stereo vision is an important method to extract 3D information from image pairs or sequences. The most common stereo configuration is a parallel setup where the camera baseline is parallel to the image planes. The image planes can also be oriented to focus on the objects for better sampling if we know approximately where the objects of interest are in the scene.

A rarely used stereo setup is called looming stereo, where the image planes are perpendicular to the baseline, as shown in Figure 1. The most important physical phenomena in looming stereo is the visual looming, which is the expansion of the size of an object projected on the retina plane when the object is moving toward the camera or the camera is moving closer to the object. Looming stereo has not received a lot of attention in the stereo reconstruction literature because of its singularity in recovering the depth along the baseline, and perhaps also due to the difficulty in matching pixels with different sizes. However, looming has been reported as an important cue for collision detection in robot navigation. Recovering depth from looming stereo is also very useful for applications such as wandering in virtual environments. In fact, looming stereo has two advantages over the conventional parallel setup. First, it has much larger overlap because the front image is almost completely seen in the back image. Next, there is less lighting variation in looming images than in parallel images. It makes correlation-based matching in looming stereo easier than in other stereo setups if we can compensate the size change.

By analyzing the visual looming phenomena, we pro-



Figure 1. Looming stereo: (a) geometric configuration; (b), (c) front and back images.

pose in this paper a disparity ratio that measures the change of pixel sizes for depth recovery from looming stereo. In looming stereo, disparity ratio plays the same role as the disparity in conventional stereo setups where the horizontal shift of corresponding pixels is used to recover the depth. Because of visual looming, adaptive window sizes have to be incorporated in the correlation stereo algorithm. Instead of minimum uncertainty principle [4], we use the maximum SNR principle that enables us to select windows with irregular shapes and sizes. Guided by the analysis of noise sensitivity, new applications of looming stereo such as view interpolation become possible.

1.1 Previous work

Though looming stereo has not been thoroughly investigated for 3D reconstruction, visual looming for obstacle avoidance has been studied by Joarder and Raviv [2] for robot navigation. A quantitative analysis of visual looming can be found in [9]. In their analysis, however, the conventional disparity measurement (*i.e.*, the pixel shift in two images) is used in the looming equation.

Stereo reconstruction from multiple panoramas (*e.g.*, [5, 7]) can also be considered as a special case of looming

stereo even though they did not carefully analyze error sensitivity etc. The singular case along baseline was not considered either.

Also related to our work is adaptive window size for improved stereo matching (correlation-based or SSD). Little [6] used several predefined correlation window and choose the best window. Jones and Malik [3]used filter banks instead of a series of windows. Kanade and Okotumi [4] introduced a statistical model of disparity and searched for the optimal window by minimizing the uncertainty of disparity. The correlation window relates to the local support to increase the reliability of certain disparity assumption. An excellent survey on local support and disparity variance assumption can be found in their paper. Recently, Boykov *et al.* [1] proposed an approach to variable window size and shape based on maximum likelihood hypothesis (plausibility) testing. They estimated the disparity by maximizing the size of window.

1.2 Overview

We describe the model of looming stereo in section 2. In section 3 we analyze the quantitative error in looming stereo. The looming stereo algorithm will be discussed in details in section 4 and 5. Experimental results are presented in section 6. Finally we present our conclusions in section 7.

2 Modelling Looming Stereo

We first describe the geometric model of looming stereo. Figure 1.a depicts the looming configuration of two cameras with baseline B. An object appears bigger in the front image than in the back image. In this paper we assume a pinhole camera model. We also assume that the optical axes of the two cameras are coincident, with the front camera closer to the scene. The two cameras have parallel retinal planes and the intrinsic parameters are known. In our subsequent analysis, we will work with normalized image coordinates.

2.1 Looming

Let $\mathbf{P} = (x, y, z)^T$ be a 3D point in object space and its projections on two image planes be $\mathbf{p}_f = (x_f, y_f)^T$ and $\mathbf{p}_b = (x_b, y_b)^T$. The subscripts f and b denote *front* and *back* cameras, respectively. Then,

$$\mathbf{p}_{f} = \begin{pmatrix} x_{f} \\ y_{f} \end{pmatrix} = \frac{1}{z} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$\mathbf{p}_{b} = \begin{pmatrix} x_{b} \\ y_{b} \end{pmatrix} = \frac{1}{z+B} \begin{pmatrix} x \\ y \end{pmatrix}$$
(1)

Consider a small shallow object that lies on the plane parallel to the image plane with the centroid at **P**. The object size is $dx \times dy$. We assume that the depth variation of the object is small enough compared to the depth of its centroid. Therefore,

$$\begin{pmatrix} dx_f \\ dy_f \end{pmatrix} = \frac{1}{z} \begin{pmatrix} dx \\ dy \end{pmatrix}$$
$$\begin{pmatrix} dx_b \\ dy_b \end{pmatrix} = \frac{1}{z+B} \begin{pmatrix} dx \\ dy \end{pmatrix}$$
(2)

where $dx_f \times dy_f$ and $dx_b \times dy_b$ are the pixel sizes of **P** in front and back images, respectively.

From equations (1) and (2), it can be shown that

$$L = \frac{x_f}{x_b} = \frac{y_f}{y_b} = \frac{dx_f}{dx_b} = \frac{dy_f}{dy_b} = \frac{z+B}{z}$$
$$z = \frac{B}{L-1}$$
(3)

where L represents the ratio between the coordinates of the pixels on the front and back images. It also represents the ratio between the sizes of pixels. We call L **disparity ratio** because the depth is inversely proportional to L - 1. Recall that in parallel stereo, the depth is inversely proportional to the disparity that is defined as the difference between the horizontal coordinates of two corresponding pixels, if the images are rectified.

Equation (3) is called the *looming equation* and has a few important properties. Looming stereo is sensitive to the quantitative error and position noise when either $L \rightarrow 1$ or $(x_b, x_b)^T \rightarrow (0, 0)^T$. Details are given in section 3. Because the looming equation can be applied to x and y dimensions independently, we will discuss the looming stereo with x dimension only in the remainder of this paper.

From looming equation (3) we also get a constraint on disparity ratio

$$L \ge 1, \|x_f\| \ge \|x_b\|, \|y_f\| \ge \|y_b\|$$
(4)

Unlike the conventional parallel stereo configuration, equation (4) provides a useful spatial constraint on the search area for the matching algorithm. The correspondences should be matched from the front image to the back image, not the other way around.

2.2 Stereo under Looming configuration

This subsection describes looming stereo, in which the disparity ratio is used to recover the depth of scene. From equation (3), we know there are two ways to depth recovery under the looming configuration. One is by the correspondences $(L = \frac{x_f}{x_h})$, as

$$z = \frac{B}{L-1} = \frac{B}{\frac{x_f}{x_b} - 1}$$
(5)

Equation (5) is actually the depth from disparity method used in the parallel stereo. As we will see in section 3, this method is very sensitive to the noise and tend to be singular near the epipoles.

The other is by size of the projected areas, as,

$$z = \frac{B}{L-1} = \frac{B}{\frac{dx_f}{dx_h} - 1} \tag{6}$$

3 Quantitative Error Analysis

In this section we discuss the noise sensitivity of looming stereo. The source of noise in looming stereo is due mainly to pixel quantization, image noise and optical distortion.

3.1 Disparity based stereo

To analyze the error property of looming stereo reconstruction algorithm described by equation (5), we take the front view as the reference image and the baseline is known. That means the coordinates in the front image and baseline is noise free. The estimated depth error is mainly determined the image coordinates of the corresponding points on the back view. The new depth, absolute error and relative error in depth estimation due to small disturbances in the back view image coordinates can be computed as following

$$z' = \frac{x_b'B}{x_f - x_b'} = \frac{(x_b + \Delta x_b)B}{x_f - (x_b + \Delta x_b)}$$
$$\Delta z = \frac{x_f B}{(x_f - x_b)^2} \Delta x_b$$
$$\epsilon_z = \frac{x_f}{x_f - x_b} \epsilon_{x_b}$$
(7)

where Δx_b is the additive position noise including quantization noise, position noise and image noise, ϵ_z and ϵ_{x_b} are the relative errors in estimated depth and back view coordinates, respectively.

Several observations can be made from (7).

- When $x_f \rightarrow x_b$ and $x_f \neq 0$, looming becomes an ill-posed problem because the error in image position affects the depth estimate significantly.
- When $x_f \rightarrow x_b$ with $x_f = 0$, looming becomes a singular problem because we cannot compute the depth in (3).
- For large $x_f x_b$ and small x_f , computing depth is robust.
- Error in baseline results in the same relative error on estimated depth. If the position error in baseline is white noise, we will get better estimate of depth with longer baseline.

Figure 2 illustrates the theoretical error bounds for looming stereo, assuming quantization error is a uniform distribution with zero mean noise, camera baseline 1 m, focal length 440 pixels and quantization error ± 0.5 pixel. A 3D point lies on the plane 4 times of camera baseline. From Figure 2, we know that the relative position error of estimated depth due to noise will be reduced significantly as the object moves away from the epipole.

3.2 Disparity ratio based looming stereo

For the disparity ratio based looming stereo approach, we can do the same error analysis to equation (6). We assume the projected area on the front view and and baseline is noise free. The noise in the projected area on the back view is the main source of the estimated depth error. By add a small turbulence Δdx_b to dx_b in equation (6), we get

$$z' = \frac{B}{\frac{dx_f + \Delta dx_f}{dx_b + \Delta dx_b} - 1}$$

$$\Delta z = \frac{dx_f B}{(dx_f - dx_b)^2} \Delta dx_b$$

$$\epsilon_z = \frac{dx_f}{dx_f - dx_b} \epsilon_{dx_b}$$
(8)



Figure 2. Relative error in depth caused by quantization error. The focal length is 440 pixels, the camera baseline is 1m, and the depth is 4 m.



Figure 3. Foreshortening in both looming and parallel stereo. Clearly there will be more seriously foreshortening in parallel stereo than in looming stereo.

Note that the size of projected areas dx_f and dx_b are not related to the projected position x_f and x_b . That means depth recovery by disparity ratio (equation 6) is much more robust than by disparity (equation 5) with $x_f \rightarrow 0$.

It is worth noting that foreshortening is less significant in looming stereo than in conventional parallel stereo. Referencing to Figure 3, with the same camera baseline, there will be $\alpha \ge \beta$, where α and β is the view angle difference of the same 3D point viewed in parallel stereo and looming respectively. It means that the foreshortening in looming stereo is relative smaller than in parallel stereo. Because resulting in little intensity changing, it also benefits the matching between two images.

Correlation is a basic technique for matching image points. While correlation methods are often the preferred for stereo matching, they do have some shortcomings. In next section we will discuss how to select optimal correlation window sizes for stereo matching. Adaptive window sizes are particularly important for looming stereo because of significant size change of object appearance in the front and back images.

4 Optimal Window for Intensity Correlation

The shape and size of correlation window will significantly affect the disparity estimates. If the window size is too small, we cannot include enough image intensity variation. Correlation will be sensitive to image noise and there would be many correlation peaks along the epipolar line. If the window size is too large, we cannot avoid the effects of perspective distortion and disparity variance. Window shape also plays an important role for accurate estimation of disparity [1]. Window size is especially critical when there is disparity discontinuity (*e.g.*, around an object boundary).

Unlike the conventional approach, we consider the disparity variance and intensity variance to affect the result of correlation in the same manner because they both introduce noise into correlation. We treat these two kinds of noise uniformly as an additive correlation noise. Furthermore, we do not assume the distribution of correlation noise [4], but rather, estimate it from the images themselves. This is an important distinction: Because the disparity is the output of stereo reconstruction, we should not assume the disparity distribution prior. Otherwise, we can only get the optimal depth estimates under a predefined disparity distribution.

This is why we formulate the disparity estimation as a problem of detecting signals from a given set of candidates by searching for correlation peaks.

Let $I_0(x, y)$ and $I_1(x, y)$ be two images of the same scene, $I_0(x_0, y_0)$ and $I_1(x_1, y_1)$ represent the projections of the same 3D point. We assume that and are equal in intensity except for the additive noise n(x, y). The correlation between $I_0(x_0, y_0)$ and $I_1(x_1, y_1)$ is given by

$$C = \frac{1}{N} \sum_{(u,v) \in w} \left\{ I_0(x_0 + u, y_0 + v) I_1(x_1 + u, y_1 + v) \right\}$$
(9)

where w is the correlation window, and N is the number of pixels in the window.

The signal to noise ratio (SNR) of the correlation is defined as

$$SNR(x, y) = \frac{C^{2}(x, y)}{E\left\{n_{c}^{2}(x, y)\right\}}$$
$$n_{c}(x, y) = \frac{1}{N} \sum_{(u, v) \in w} \{I_{1}(x_{1}+u, y_{1}+v)n(x_{0}+u, y_{0}+v)\}$$

where $E\{n^2(x, y)\}$ is the power of output noise, and $n_c(x, y)$ denotes the correlation noise, which is the output noise of correlation. In signal detecting theory, SNR is related to the probability of detection (PD). Maximizing SNR means maximizing the probability of detection for a given probability of false alarm (PFA) in the Neyman-Pearson sense, or minimize PFA for given PD. Consider the following two hypotheses:

$$egin{aligned} H_0 &: n_c(x,y) \ H_1 &: \widetilde{C}(x,y) + n_c(x,y) \end{aligned}$$

where $\tilde{C}(x, y)$ is the correlation of the same intensity pattern in both images. $n_c(x, y)$ is the correlation between and correlation noise. To simplify the analysis, we assume is a zero-mean Gaussian noise with variance σ . The conditional probabilities of C(x, y) under both two hypotheses are:

$$p(C(x,y)|H_0) = \frac{1}{\sqrt{\pi\sigma}} exp\left\{-\frac{C^2(x,y)}{2\sigma^2}\right\}$$
$$p(C(x,y)|H_1) = \frac{1}{\sqrt{\pi\sigma}} exp\left\{-\frac{[C(x,y) - \widetilde{C}(x,y)]^2}{2\sigma^2}\right\}$$



Figure 4. Probability of false alarm (PFA) and probability of detection (PD) versus SNR in maximum likelihood (ML) test.(a) Maximum likelihood test;(b) PD and PFA of maximum likelihood test

We take maximum likelihood (ML) test for these two hypotheses with no prior bias in favor of either H_0 or H_1 . Then H_1 holds if and only if

$$p(C(x,y)|H_0) < p(C(x,y)|H_1)$$
(10)

The ML test is illustrated in Figure 4a. The probability of false alarm (PFA) is defined as

$$p_{fa} = \int_{\frac{\widetilde{C}(x,y)}{2}}^{\infty} p(C(x,y)|H_0) dC(x,y)$$

$$= \int_{\frac{\widetilde{C}(x,y)}{2}}^{\infty} \frac{1}{\sqrt{\pi\sigma}} exp\left\{-\frac{C^2(x,y)}{2\sigma^2}\right\} dC(x,y)$$

$$= 1 - \Phi(\frac{\sqrt{SNR - 1}}{2}) \qquad (11)$$

where $\Phi(x)=\int_x^\infty \frac{1}{2\pi} exp[-\frac{x^2}{2}]dx$. Similarly, probability of detection (PD) is defined as

$$p_d = \int_{\frac{\widetilde{C}(x,y)}{2}}^{\infty} p(C(x,y)|H_1) dC(x,y)$$
$$= \Phi(\frac{\sqrt{SNR-1}}{2})$$
(12)

The relationship between PFA and SNR is shown in Figure 4b. As shown in Figure 4, higher means higher probability of detection and lower probability of false alarm, therefore, more reliable decision. Therefore, we adopt maximizing SNR as the optimal principle for adaptive correlation windows.

If n(x, y) is the white noise, the optimal correlation window size is obviously the largest window size. Unfortunately, because the correlation noise includes portions of the image noise, projective distortion noise and mismatch noise, it cannot be modeled as the white noise. Instead, it should be estimated together along with the correlation, because the intensity variance in an image results from all these three sources. For example, if there is no texture (equal in intensity), geometry projective distortion will not affect the intensity variance nearby the projected position.

In our approach, the optimal window w_{opt} is chosen so that

$$w_{opt} = \arg\max\left\{SNR(x,y)\right\} \tag{13}$$

The best window provides the optimal local support with maximum probability of detection.

Consider the problem of estimating the disparity ratio at a given point. Because the true variance σ_{xy} is not available, an estimated variance $\hat{\sigma}_{xy}$ is used to estimate the optimal correlation window. We estimate the power of correlation noise by

$$\hat{\sigma}_{xy} = E\left\{n^2(x,y)\right\} = E\left\{\left[I_f(\frac{x}{L},\frac{y}{L}) - I_b(x,y)\right]^2\right\} (14)$$

where L is the disparity ratio. After the correlation is computed, we expand the window by adding more pixels that satisfy (13).

5 Looming Stereo with Adaptive Windows

5.1 Optimal correlation window size

Both visual looming and maximum SNR window have to be considered when selecting adaptive window sizes in looming stereo. In practice, in order to include information of the image intensity variance and use the disparity ratio constraint (4), we first determine the maximum SNR correlation window in the front image, then compute the corresponding window in the back image using equation equation (3).

In looming stereo, adaptive window sizes can be determined as follows:

- For a given front mage point **p**_f and a potential disparity ratio we estimate power of the correlation noise by (14).
- Set initial correlation window w_f for the pixel (x_f, y_f) in the front image.
- Expand the correlation window w_f in each direction. A new pixel is accepted if and only if the SNR function is non-decreasing after the expansion.
- For any point (x, y) ∈ w_f and disparity ratio L(x, y), compute the correspondence in the back image. These correspondence points in the back image form the correlation window w_b.
- Compute the correlation with the estimated windows.

5.2 Estimate Disparity Ratio

To solve disparity ratio along the epipolar line, we use dynamic programming with the cost function defined as the sum of correlation along the epipolar line. For a pair of corresponding epipolar lines on the front and back images, the cost function of disparity ratio is defined as

$$\begin{split} Cost(x,y,L) &= \sum_{(x,y)} C(x,y,L(x,y)) \\ &= \sum_{xy} \sum_{(u,v) \in W_f(x,y))} \left\{ I_b(x+u,y+v) I_f(\frac{x+u}{L(x,y)},\frac{y+v}{L(x,y)}) \right\} \end{split}$$

(15)

where $W_f(x, y)$ denotes the optimized correlation window, L(x, y) the disparity ratio estimate of the image point $\mathbf{P}_f(x, y)$, $I_b(x, y)$ the estimated intensity at pixel (x, y) in the back image. The cost function is separable. Our looming stereo problem can be expressed as maximizing the cost function defined by (15). We solve it using dynamic programming similar to [8].

6 Experimental Results

In this section, we describe experiments using images of synthetic and real scenes.

6.1 Synthetic scene

Figures 1(b,c) show a pair of looming stereo images of a textured cube. Both epipoles are at the center of image. The disparity ratio range of this looming stereo pair is between 1.1 and 2.0. Figure 5(a) shows the depth map computed by a standard correlation algorithm with a fixed size window 10×10). Figure 5(b) is the plot of the depth map viewed from another viewpoint. As shown in Figure 5(a), large error occurs near the center of the image, which is the singular case of looming stereo with disparity. Figure 5(c) shows the depth map computed using our disparity ratio approach with adaptive windows, which yielded significantly better depth estimation near the epipoles. Figure 5(d) is the plot of the depth map viewed from another viewpoint. For this camera, which has a relatively small field of view (FOV), we can see that using the disparity ratio is better than using disparity, especially within the vicinity of the epipoles. Figure 5(e) shows the map of correlation window size. The brighter pixel means bigger window size (containing larger number of pixels) at that position. Figure 5(f) shows the histogram of the window size. The mean of the window size is about 127.639.

6.2 Real scene

We have applied both the disparities stereo algorithms to real looming stereo images.

Figures 6(a,b) show the images of NASA data that we taken from the optical flow test database. The camera motion between the image pairs is near looming because the epipoles is located near the center of images. The disparity based stereo algorithm with dynamic programming search method is applied to the image pair. Figure 6(c) shows the recovered depth map with conventional disparity method. Near the center of the depth map, we observe that we do not get reliable depth information. Figure 6(d) shows the depth map as a result of using the disparity ratio with adaptive windowing. In contrast, we can see that the depth estimation has improved near the center of images. Figure 6(e) shows the map of correlation window size. The brighter pixel means bigger window size (containing larger number of pixels) at that position. Figure 6(f) shows the histogram of the window size. The mean of the window size is about 94.1755.

7 Conclusion

In this paper, we have studied looming stereo where the image planes are perpendicular to the baseline. We propose to use disparity ratio to estimate the depth from looming stereo, as opposed to disparity in conventional parallel or vergent stereo. Error analysis shows that looming stereo will be divergent at the epipoles, and very sensitive to noise



Figure 5. The results of looming stereo for the "cube" image pair: Results with synthetic image: (a) Depth map by standard disparity algorithm; (b) Depth map (a) as seen obliquely, (c) Depth map by disparity ratio and adaptive windows, (d) Depth map (c) as seen obliquely, and (e) The window size map where the graylevel indicate the number of pixels in the correlation windows.(f) Histogram of (e). The mean of histogram is 127.639 and the variance is 68.57; We see that the results using the disparity ratio with adaptive windowing are better than the conventional disparity approach. This is not surprising, as the disparity ratio exhibits

near epipoles. It is robust when the image points move far away from the epipoles. This is a shortcoming for some vision task such as 3D structure recovery or depth estimate. On the other hand this error character will bring us some useful application of looming stereo, and sometimes we must deal with the looming stereo such as vision navigation in AGV, and walkthrough environment building in imagebased rendering. To compensate for the large size change in the back and front images in looming stereo, we have introduced a novel approach to computing adaptive window sizes based on the principle of optimizing signal-to-noise ratio of correlation. Optimal window shape and size are then obtained. Experimental results show that our stereo algorithm results in more accurate reconstruction than fixed window correlation, and more uniform correlation map.

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Figure 6. "Coke scene" looming stereo data set: (a) Front image; (b) Back image; (c) Depth map using disparity; (d) Depth map using disparity ratio; (e) Correlation window size map; (f) The histogram of (e)(mean = 94.1755, variance = 55.398).

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