# 3D Eigenfaces for Face Modeling 

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#### Abstract

In this paper we propose the idea of approximating an arbitrary 3D face model by a linear combination of "3D Eigenfaces". First, a set of 3D face models are constructed by a face modeling system and the principal component analysis is performed over the data set. The eigen vectors associated with the largest eigen values are extracted to form the eigen space. We call these eigen vectors "3D Eigenfaces". Any face model can then be approximated by a linear combination of these eigenfaces, thus reducing the whole space of face shape to the smaller space of these coefficients. We show that the reconstructed faces using these 3D eigenfaces are sufficiently close to the original faces. This result can be applied in image-based reconstruction of 3D face models.


## 1 Introduction

3D face modeling is of wide interest among not only researchers but also ordinary people. One may imagine that if building 3D face models is as easy as taking photographs, such service may be as popular as photographs.

Image-based 3D face modeling has been a popular topic for many years. A partial list of references include [1, 3, 4, $5,6,7,2]$. Many of the early approaches use the frontal and side views [1, 2, 4, 5]. Later approaches try to use multiple general views [3, 7].

Underlying most of these approaches is the face mesh model. Since a face is a curved surface, hundreds of triangular patches are required to represent the shape. Unfortunately, it is impossible to recover the 3D coordinates of all the vertices, because these vertices are mostly not feature points detectable from face images.

On the other hand, though face shape differs from person to person in the absolute sense, different faces are similar to each other. The number of dimensions to represent the shape space needs not to be as high as the number of vertices. To reduce the dimensions, the conventional wisdom is to collect a set of 3D face models and use principal component analysis to find the dominant dimensions.

In this paper, we show a result by such an approach.

Firstly, we built 71 face models using a semi-manual system based on frontal and side face images. Then principal component analysis is performed to extract the 15 dominant eigen vectors, which we call the 3D Eigenfaces. Any face is then projected onto these dimensions and represented by a linear combination of these 3D eigenfaces. By changing the coefficients of the linear combination, the face varies from more male-looking to more female-looking, from long faces to short faces, from narrow faces to wide faces, etc. With this set of 3D eigenfaces, the original task of determining the vertices becomes the task of determining the 15 coefficients, thus reducing the number of unknowns from over one thousand to only 15 in this case. Experiments show that this set of 3D eigenfaces is powerful enough to approximate most of the faces we tried.

Similar ideas of using linear space for 3D face modeling have been reported by Liu, et al. [7]. However, the linear space is determined by artists in their case. Though each dimension is semantically well defined, it is not clear if that space is stochastically optimal.

## 2 Calculating 3D Eigenfaces

Let a face be represented by a group of $N$ vertices in $R^{3}$ which are connected in a special way. We call the way of connecting these vertices the topology and the coordinates of these vertices the geometry. A typical number of vertices needed to realistically represent a 3D face is around 400 . This means, a face is described by a vector of dimension 1,200 , or equivalently, a point in a 1200 -dimensional space. An ensemble of faces maps to a collection of points in this large space.

The faces, being similar in the general shape, are not randomly distributed in this space, and should only cover a small low-dimensional subspace. Here we apply the conventional wisdom of principal component analysis to find the vectors in this space which best account for the distribution of face shapes. They are the eigen vectors associated with the largest eigen values, of the covariance matrix defined by the sample face set. We call these eigen vectors "3D Eigenfaces". Let the number of 3D eigenfaces be $M$.

The $M$ 3D eigenfaces define the low-dimensional subspace of face shapes. While the $M$ 3D eigenfaces have the same dimension as the original sample faces, the subspace is only a linear combination of the $M$ 3D eigenfaces, and thus, it is $M$-dimensional.

Let the sample face set be represented by $\mathbf{f}_{i}, i=1, \ldots, S$, where $S$ is the number of sample faces. The average 3D face is first computed by

$$
\begin{equation*}
\mathbf{f}_{0}=\frac{1}{S} \sum_{i=1}^{S} \mathbf{f}_{i} \tag{1}
\end{equation*}
$$

We can then construct the covariance matrix as

$$
\begin{equation*}
\mathbf{C}=\frac{1}{S} \sum_{i=1}^{S}\left(\mathbf{f}_{i}-\mathbf{f}_{0}\right)\left(\mathbf{f}_{i}-\mathbf{f}_{0}\right)^{T}=\mathbf{A} \mathbf{A}^{T} \tag{2}
\end{equation*}
$$

where $\mathbf{A}=\left[\mathbf{f}_{1}-\mathbf{f}_{0}, \ldots, \mathbf{f}_{S}-\mathbf{f}_{0}\right]$.
Matrix $\mathbf{C}$ has a dimension of $3 N \times 3 N$. We can compute the eigenvalues and corresponding eigenvectors by a program from [8]. Alternatively, if $S \ll 3 N$, we can first solve a $S \times S$ matrix problem, and then find the corresponding eigen vectors [9].

Generally, there are $S$ non-zero eigenvalues. However, since the sorted eigenvalues decreases quickly, we can keep the first largest eigenvalues only and abandon the rest while still being able to approximate the subspace sufficiently well.

## 3 Representing an Arbitrary Face by Linear Combination of 3D Eigenfaces

Let the number of selected largest eigenvalues be $D$ and the corresponding eigenvectors be $\mathbf{e}_{i}, i=1, \ldots, D$.

Assuming that the sample faces are representative of the face space, we can now claim that the resulting 3D eigenfaces are sufficient to represent the subspace so that an arbitrary face $\mathbf{f}$ can be represented by a linear combination of the 3 D eigenfaces,

$$
\begin{equation*}
\mathbf{f} \approx \mathbf{f}_{0}+\sum_{i=1}^{D} c_{i} \mathbf{e}_{i} \tag{3}
\end{equation*}
$$

where $c_{i}$ 's are the coefficients for the 3D eigenfaces, which can be determined by

$$
\begin{equation*}
c_{i}=\left(\mathbf{f}-\mathbf{f}_{0}\right)^{T} \mathbf{e}_{i} \tag{4}
\end{equation*}
$$

The error vector due to the use of the truncated subspace is defined by

$$
\begin{equation*}
\mathbf{d}=\mathbf{f}-\mathbf{f}_{0}-\sum_{i=1}^{D} c_{i} \mathbf{e}_{i}=\sum_{i=D+1}^{S} c_{i} \mathbf{e}_{i} \tag{5}
\end{equation*}
$$

And the error can be quantitized by its norm $\|\mathbf{d}\|$. Since coefficients after the $(D+1)$-th are small, the error is small.

We will show in Section 5 how the face shape changes by varying the coefficients. Note that this representation can also be used for person identification by comparing the coefficients $c_{i}$ 's of an input face with the coefficients of all the faces in the database.

## 4 Determining the Coefficients from a Subset of Vertices

Recall that the objective here is to reconstruct a face model from images. If we knew all the vertices, we would not have to think about the 3D eigenfaces. Suppose that we somehow only know a subset of the vertices. We are interested in knowing how many vertices are needed to correctly determine the coefficients and thus the face model itself.

Let the number of known vertices be $N^{\prime}$. We can construct a subspace of $3 N^{\prime}$ dimensions. Let the vector representing the $N^{\prime}$ vertices be $\mathbf{f}^{\prime}$. It is intuitive that the coefficients satisfying

$$
\mathbf{f}=\mathbf{f}_{0}+\sum_{i=1}^{D} c_{i} \mathbf{e}_{i}
$$

also satisfies

$$
\begin{equation*}
\mathbf{f}^{\prime}=\mathbf{f}_{0}^{\prime}+\sum_{i=1}^{D} c_{i} \mathbf{e}_{i}^{\prime} \tag{6}
\end{equation*}
$$

where $\mathbf{f}^{\prime}, \mathbf{f}_{0}^{\prime}, \mathbf{e}_{i}^{\prime}$ are respectively vectors for the $N^{\prime}$ vertices.
Given these vectors, we can determine $c_{i}$ 's in a similar way as (4). Once $c_{i}$ 's are obtained, we use them to reconstruct the face in the full dimension.

## 5 Experimental Results

In this section we report the experimental results. At first a set of 71 face models are reconstructed by FaceFit-RS-, a system for manually reconstructing a 3D face model from a frontal and a side views of the face. FaceFit-RS- is modified from FaceFit [10], a system sponsored by IPA of the Japanese government. FaceFit can only be modified using a frontal view. We modified the system so that the face model can be deformed based on a side view as well. One of the examples is shown in Fig. 1. The face model seen from a new view point is shown in Fig. 2.

We collected 71 such models, and a mean face model (Fig. 3) is first computed using these 71 models. Then we computed the eigenvalues and eigenvectors from the covariance matrix.

The sorted eigenvalues are shown in Fig. 4. By looking at the graph, we decided to use the first 15 eigen values and


Figure 1: Fitting a polygon model to a frontal face image by FaceFit -RS-


Figure 2: Face model seen from a new viewpoint


Figure 3: The mean face
associated vectors. These eigenvectors are the so-called 3D Eigenfaces.


Figure 4: Eigenvalues

### 5.1 The 3D Eigenfaces

In the following, we show how a face is gradually better represented by the 3D eigenfaces as the number of 3D eigenfaces increases.

We show 2 shots of the sequence. Figures 5 and 6 respectively show two faces using the first six 3D eigenfaces and using all the fifteen 3D eigenfaces. It can easily seen that the second one is better than the first one.


Figure 5: Face reconstructed from the first 7 eigenfaces

In the following figures 7-10, we show how each 3D eigenface changes the face shape. Each face shown is the sum of the mean face and a 3D eigenface exaggrated to a positive direction or a negative direction by setting the coefficients for the 3D eigenfaces to a positive or negative values.

We have developed a Windows-based system to change the face shape in realtime by changing the coefficients for the 3D eigenfaces. The toolbars are shown in Fig. 11.


Figure 6: Face reconstructed by all the 15 3D eigenfaces


Figure 7: First coefficient set to 3


Figure 8: First coefficient set to -4


Figure 9: Third coefficient set to 2


Figure 10: Third coefficient set to -2


Figure 11: Changing a textured face model by changing the coefficients of the 3D eigenfaces

### 5.2 Determining the Coefficients

To investigate how many vertices are required in order to determine the coefficients within a satisfactory error bound, we prepared 6 data sets, with each set having 15, 20(Fig.12), 25, 30(Fig.13), 35 and 40(Fig.14) vertices shown by black dots out of the original 409 vertices.


Figure 12: The 20 vertices used


Figure 13: The 30 vertices used

These subsets of vertices are used to compute the coefficients for the 3D eigenfaces. And using these coefficients we then computed corresponding face models. They are shown in Fig.15, Fig.16, and Fig. 17.

It can be easily seen that the more number of sample vertices we use, the closer the reconstructed face model is to the original face. However, the improvement is very slow after the number of sample vertices is more than 20 . We define the difference between the original model and the reconstructed model as the square root of the sum of squared Euclidean distances between the corresponding vertices on the original model and the reconstructed model. The relation between the number of input points and the difference is shown in Fig. 18.


Figure 14: The 40 vertices used


Figure 15: Face reconstructed from the 20 vertices


Figure 16: Face reconstructed from the 30 vertices


Figure 17: Face reconstructed from the 40 vertices

## 6 Conclusions



Figure 18: The difference decreases as the number of vertices used increases.

In this paper we have proposed to represent the space of 3D face shapes by the so-called 3D Eigenfaces, which are the eigenvectors associated with the largest eigenvalues of the covariance matrix constructed from sample 3D face models. These 3D eigenfaces represent a linear subspace of the original space of a much larger dimensions. With these eigenfaces, an arbitrary face model can be represented by merely a linear combination of these 3D eigenfaces, thus reducing the problem of determining the positions of all the vertices in the mesh model to the problem of determining the coefficients of the linear combination. We also show that using only a smaller percentage of the vertices of known 3D coordinates can determine the coefficients precisely enough. Experimental results confirm all these points. We are developing a system of image-based face modeling based on the key idea of 3D eigenfaces.

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