Structural Analysis of On-Line Curves Based on Topological Turning Patterns

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Abstract

Compared with machine-printed characters, handwritings have variety of shape deformations. One of the important goals of character recognition is to find some qualitative features that are invariant under deformation of shapes. In this paper, we propose a method for structural analysis of on-line handwritten curves based on topological turning patterns. The topological turning pattern is described by initial direction, directional change, and inflection number of the curve, computed from directional features of the segments that constitute the curve. The mathematical properties of the topological turning pattern is explained along with the experimental results of the method on numerals.

1 Introduction

Structural pattern recognition attempts to describe objects using mathematical models suitable for further processing. The basic idea is that a complex pattern can be described recursively in terms of simpler subpatterns [1]. Because the models are clearly described, it has a few attractive properties: it does not require a large amount of training data, recognition performance on new data is guaranteed in contrast to the "black box" approach in which the function from the input to the output is unclear, and the number of features used to describe a class of pattern may vary from one class to another.

Nishida [2] proposed an algebraic description of the curve structure that tries to integrate local features (line segments or primitives) into global features (primitive sequence label or PS-label in their term) that are invariant in the number of primitives that constitute a primitive sequence. The biggest advantage of their method is that extremely "clear" algebraic description is obtained in the high level by applying operations to objects on the lower level. However, since the method was developed for off-line processing of handprinted characters, it contained several problems in order to be applied to on-line processing of handwritings. The major problems on-line users encounter is that temporal information which is crucial in on-line processing is not properly incorporated in their description system. In their scheme, the head and tail of a segment is determined by the coordinate values of the points and have nothing to do with the time sequences of stroke end points— an inconsistency with the common descriptive custom in on-line handwriting analysis.

This paper presents a new structural description method of on-line curves that resolves all the above problems. In this method, a single curve is analyzed into a single compact representation regardless of the winding complexity. The segments of a curve are concatenated always at its head or tail without overlapping.

Section 2 provides a brief explanation of structure of a single curve, while Sec. 3 discusses the relation of multiple curves. Section 4 describes an application of our description method to character recognition, showing the experimental results on numerals.

2 Structure of a Curve

The set of points defining a curve generally results in large volume of data and it is necessary to express the curve in a more compact way, yet without any loss of significant information. Linear piecewise polygonal approximation [3] is the most frequently used one that produces a polygon which closely resembles the original curve. In this research, a polygonal approximation method based on the split and merge method is adopted for obtaining a piecewise linear curve that fits to the original curve within an acceptable error range. Henceforth, our discussions will be restricted to piecewise linear curves. For convenience we shall usually omit the words "piecewise linear," and whenever we speak of "curves" we mean piecewise linear curves.

2.1 Structure of line segments

A curve consists of a set of line segments (simply called *segment* from now on) concatenated in sequence. When a segment s runs from a point t to a point h, the point h is called the *head* of the segment, and t the *tail*. The points that correspond to a head or tail of some segments are called *knots*.

In order to simplify formulas and thereby clarify ideas, we shall use some notations. A segment s that has knots h and t as its head and tail is denoted as $s : t \mapsto h$. A single segment may constitutes a curve. However, most curves consist of two or more segments. The curve formed by only two segments has simplest non-trivial structure, and we shall call such curve a *basic arc*. When a basic arc c is formed by concatenating two segments s and t, c is represented in terms of s and t as follows:

$$c : s \longrightarrow t \tag{1}$$

In the above "concatenation" notation, the object in the tail side of the arrow is assumed to be generated before the object in the head side. The curve is formed by unifying the head of the object in the tail side of the arrow and the tail of the object in the head side of the arrow.

Remark 1. Note that each segment of a curve is directional, and no two consecutive segments in a curve are collinear. For a segment $s : (t_x,\ t_y) \mapsto (h_x,\ h_y)$, we shall denote by $\vec{s}$ the vector $(h_x - t_x,\ h_y - t_y)$ which we call the *segment vector* of s.

The direction of a segment is classified into eight classes. For a segment s, if θ is the angle between $\vec{s}$ and the positive x-axis, the directional index i of s is defined as follows:

$$\begin{aligned}
1)\ i &= 1 \text{ if } \theta > -\delta \text{ and } \theta \le \delta \\
2)\ i &= 2 \text{ if } \theta > \tfrac{1}{4}\pi - \delta \text{ and } \theta \le \tfrac{1}{4}\pi + \delta \\
3)\ i &= 3 \text{ if } \theta > \tfrac{1}{2}\pi - \delta \text{ and } \theta \le \tfrac{1}{2}\pi + \delta \\
4)\ i &= 4 \text{ if } \theta > \tfrac{3}{4}\pi - \delta \text{ and } \theta \le \tfrac{3}{4}\pi + \delta \\
5)\ i &= 5 \text{ if } \theta > \pi - \delta \text{ and } \theta \le \pi + \delta \\
6)\ i &= 6 \text{ if } \theta > \tfrac{5}{4}\pi - \delta \text{ and } \theta \le \tfrac{5}{4}\pi + \delta \\
7)\ i &= 7 \text{ if } \theta > \tfrac{3}{2}\pi - \delta \text{ and } \theta \le \tfrac{3}{2}\pi + \delta \\
8)\ i &= 8 \text{ if } \theta > \tfrac{7}{4}\pi - \delta \text{ and } \theta \le \tfrac{7}{4}\pi + \delta
\end{aligned} \tag{2}$$

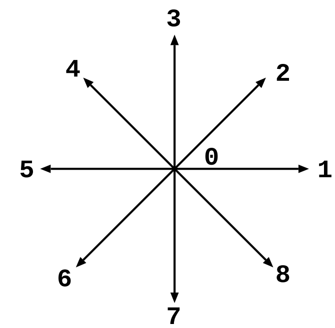

Figure 1: All the types and the corresponding indices of direction used in our description system.

where, $\delta = \pi/8$. All the types of direction used in our description are illustrated in Fig. 1.

Definition 1 (Direction of a segment) *Let s be a segment. The number $dir(s) \in \{1, \cdots, 8\}$ calculated by (2) is called the direction of s.*

It is possible to associate a sign to a basic arc according to its orientation. If we turn counterclockwise in traveling the arc from the first to the last segment, we shall call the orientation *positive orientation*, and the other one *negative orientation*.

Definition 2 (Orientation of a basic arc) *For a basic arc $a : s \longrightarrow t$, the orientation of a, $ori(a)$, is decided by the sign of $det(\vec{s},\ \vec{t}) = \begin{vmatrix} \vec{s}_x & \vec{s}_y \\ \vec{t}_x & \vec{t}_y \end{vmatrix}$ as*

$$ori(a) = \begin{cases} 1 & : \ det(\vec{s},\ \vec{t}) > 0 \\ -1 & : \ det(\vec{s},\ \vec{t}) < 0 \end{cases} \tag{3}$$

Remark 2. The orientation of an open curve is derived from the orientation of its basic arcs if and only if all the basic arcs have uniform orientation. In that case, the orientation of the curve is defined by the orientation of one of its basic arcs.

Now, we want to measure how much a segment turns away from the direction of its predecessor.

Definition 3 (Directional change of a basic arc) *Let $a : s \longrightarrow t$ be a simple arc, and point p the knot commonly possessed by s and t. We define the curvature of a at p and the directional change of a as*

$$cur_a(p) = \begin{cases} mod(dir(t) - dir(s) + 8,\ 8) \\ mod(dir(s) - dir(t) + 8,\ 8) \end{cases} \tag{4}$$

$$dc(a) = ori(a) \cdot cur_a(p). \tag{5}$$

If $cur_a(p) = 0$ for every knot p in a curve a, a is a straight line. If $cur_a(p) \geq 2$ for some knot p, a makes an acute angle at p and p is called a *corner point.* In a basic arc $a : s \longrightarrow t$, if the value of $dc(a)$ is known, $dir(s)$ can be calculated from $dir(t)$, and vice versa, by the formula

$$\begin{aligned} dir(t) &= mod(dir(s) + dc(a) + 7,\ 8) + 1 \\ dir(s) &= mod(dir(t) - dc(a) + 7,\ 8) + 1 \end{aligned} \quad (6)$$

which we shall indicate with the notations

$$\begin{aligned} dir(t) &= dir(s) \oplus dc(a) \\ dir(s) &= dir(t) \ominus dc(a). \end{aligned} \quad (7)$$

A useful expression for a general curve $c : s_0 \longrightarrow^* s_n$ can be obtained by successively applying the formula (7) on the basic arcs of a curve:

$$dir(s_n) \quad = \quad dir(s_0) \oplus \sum_{i=1}^{n} dc(s_{i-1} \mapsto s_i). \quad (8)$$

Observing the formula (8) reveals that the term $\sum_{i=1}^{n} dc(s_{i-1} \mapsto s_i)$ is the directional change of c and can be simply expressed by $dc(c)$. Thus, the definition of directional change can be extended from basic arc to curve:

Definition 4 (Directional change of a curve) *For any curve $c : s_0 \longrightarrow^* s_n$, it's directional change is defined as the sum of the directional changes of every basic arc embedded in c:*

$$dc(c) = \begin{cases} \sum_{i=1}^{n} dc(s_{i-1} \mapsto s_i) \\ (\sum_{i=1}^{n} dc(s_{i-1} \mapsto s_i)) + dc(s_n \mapsto s_0) \end{cases} \quad (9)$$

The sequence of $dc(s_i \mapsto s_j)$'s which appear in the formula (**??**) characterizes how a curve turns on the plane and will be called *turning pattern* of the curve.

Remark 3. In a curve there may exist two or more segments that have the same direction and appear in a sequence. We call such segment sequence *straight arc.* Owing to straight arcs, the turning pattern obtained by the definition given above can contain 0's, which are extraneous information in calculating the directional change of the whole curve. Therefore, as a step of abstraction we can eliminate 0's from turning pattern and make a new compact turning pattern which we call *topological turning pattern.* The geometric interpretation of the topological turning pattern of a curve is to take every straight arc in the curve and shrink it into one segment fixing the two end points of the arc. The curve created according to a topological turning pattern is called a *topological curve.* Note that the topological curve contains no straight arc but preserves the turning characteristic of the original curve.

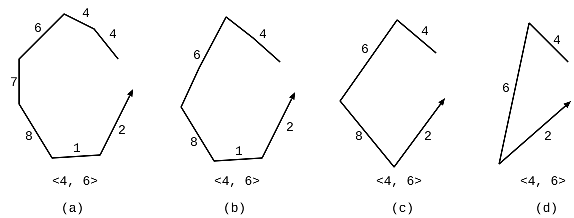

Figure 2: Instances of idr-dc equivalent curves.

2.2 First abstraction of the shape of curves

We shall now take the first step in algebrization of the shape of curves. We introduce two fundamental properties of a curve that allows differentiation of intuitively different curves. First, we give the measure a name:

Definition 5 (idr-dc characteristic) *For any curve c, $X^2(c) = < idr(c),\ dc(c) >$ is the idr-dc characteristic. idr and dc denote the initial direction ($= dir(s_0)$) and the directional change, respectively.*

Based on the idr-dc characteristic, other many useful properties can be computed. Therefore, we call the *idr* and *dc* properties *fundamental* properties, and the other properties derived from the fundamental properties *derived* properties. For example, the final direction of curves is a derived property, as will be explained below.

Example 1. Instances of curves with the idr-dc code are illustrated in Fig. 2. Notice that the number of in-between strokes vary but the shapes maintain similarity by possessing the same idr-dc values. In other words, X^2 value of a curve is invariant under "sub-divisional variations" of the curve illustrated in Fig. 2.

When two curves c_1 and c_2 have same idr-dc characteristic, we say they are idr-dc equivalent. The summary of arguments in this section is formalized by the following theorem:

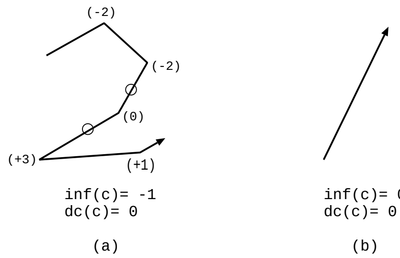

Figure 3: Instances of curves with inf codes. Circled segments are inflection segments. Numbers written besides knots enclosed in parenthesis are directional changes.

Theorem 1 *Any two curves c_1 and c_2 have the same initial and final direction iff they are idr-dc equivalent.*

2.3 Second abstraction of the shape of curves

The dc-characteristic of a curve measures only the total turning angle of the curve as a whole, and does not check the change of orientations of the embedded arcs. Therefore it cannot differentiate the symbols '−' and 'Z' or 'C' and 'ε'. In order to check the winding characteristic of a curve properly, we introduce a new measure:

Definition 6 (Inflection index) *Let c and T(c) = $dc_1 dc_2 \cdots dc_n$ be a curve and its turning pattern. Without loss of generality, we can assume that c does not contain straight arcs. The inflection index of c is defined as*

$$\begin{aligned} inf(c) &= sn \text{ where,} \\ s &= ori(dc_1) \\ n &= \text{the no. of } i \ (2 \le i \le n) \ , \ dc_{i-1}dc_i < 0 \end{aligned}$$

The segments s_i for which $dc(s_{i-1} \mapsto s_i)dc(s_i \mapsto s_{i+1}) < 0$ are called the inflection segments.

Example 2. Instances of curves with the inf code are illustrated in Fig. 3. Notice that the curves (a) and (b) have same dc values but vary in inf numbers.

Definition 7 (idr-dc-inf characteristic) *For any curve c, $X^3(c) =< idr(c),\ dc(c),\ inf(c) >$ is the idr-dc-inf characteristic. inf(c) is the inflection number of c.*

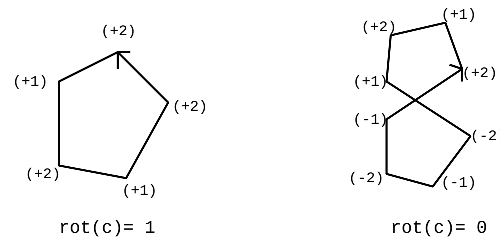

Figure 4: Examples of closed curves with their rotation indices. The arrow segments indicate the final segments.

2.4 Closed Curves

In a closed curve, the initial segment is not noticeable and *idr* does not have much to do with the whole shape of the curve. Therefore, we set a convention that $idr(c) = 0$ for any closed curve c. Closed curves have the remarkable property that their directional changes are 8 multiplied by an integer (*cf.* [4, 5]).

Definition 8 (Rotation index of a closed curve) *The rotation index of a closed curve c is defined as $rot(c) = dc(c)/8$.*

The rotation index of a curve is a derived property because it can be computed from $dc(c)$ which is a fundamental property.

Theorem 2 (Rotation index of a closed curve) *For any closed curve c, $rot(c) = \pm n$, where the sign coincides with the natural orientation of the curve.*

In Fig. 4 are some examples of curves with their rotation indices. Observe that the rotation index changes sign when we write the curves in opposite direction. The rotation index of closed curves is a topological invariant [6].

In closed curves, the rotation index is a useful property which distinguishes topologically different closed curves. For instance, as illustrated in Fig. 4, the symbols '0' and '8' can be distinguished by their rotation indices.

3 Multiple Points

In this section, we describe *multiple points* that result from intersection of curves.

3.1 Structure of multiple points

A segment of a curve may intersect another segment of the same curve or of different curve to generate a multiple point. The number and types of multiple points is an important property to specify in describing curves.

Nishida [2] classified singular points into X, K, and T type according to the adjacent structure of two curves on the singular point, and described the structure of a singular point by binary relation of the two curves. In the real world of handwritings, however, more than two curves may meet in a multiple point in a complex way, and it is not always possible to analyze a singular point into one of X, K, and T types.

In our description, there are three basic types in which two segments meet to produce a multiple point: X (crossing), T (touch), or L (join). Notice that according to our description Nishida's K type can be looked upon as a special case of T type in which two segments touch the same point on a segment. Coinciding end points are regarded as multiple points, i.e., we define the segments to be closed.

3.2 Description of multiple points

We describe a multiple point by the relation of the multiple point and all the segments which contain the point. From the view point of the multiple point, every intersection of a segment with the point is either "in the middle (M)" or "at the extremity (E)." The structure of a multiple point p is described by n-ary relation of n curves meeting at p.

$$p : [e_1(a_1), \cdots, e_i(a_i); m_1(b_1), \cdots, m_j(b_j)] \qquad (10)$$

where, e_k $(k = 1, 2, \cdots, i)$ are segments having p at the extremity, $a_p \in \{h, t\}$, and m_l $(l = 1, 2, \cdots, j)$ are segments having p in the middle, $b_q \in \{i, f, m\}$. The order of p defined as the number of edges incident to p is calculated as

$$order(p) = i + 2j. \qquad (11)$$

4 Experiment

The primary information we use for differentiating numerals is the idr-dc-inf characteristic of the shapes. The analysis is undertaken hierarchically from the original strokes to the higher level information.

Writers need to know when and how the system succeeds or fails in analyzing the structure of curves. Visualizing curves structure in a 3D landscape may offer insights into the nature of the underlying logic and data base the system operates. We display the analysis procedure in an intuitive way by incrementally overlaying the features analyzed and the data objects processed at each consecutive step to clarify the interrelationship of the objects (Fig. 5).

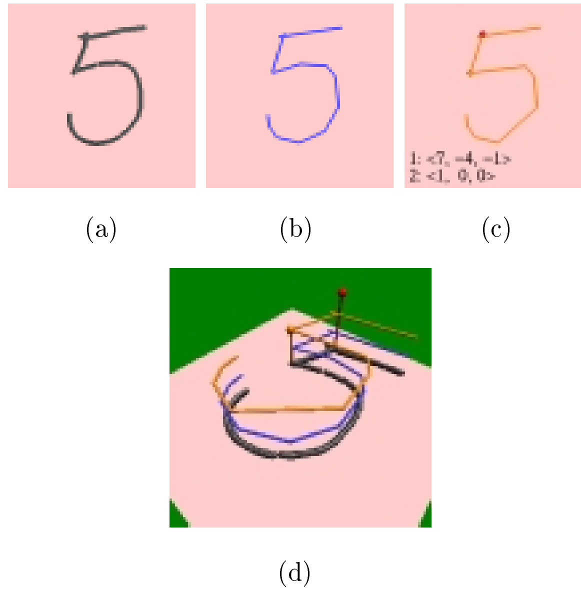

Figure 5: Geometric objects visualized in 2D or 3D: (a) original strokes, (b) polygonally approximated curve, (c) topological curve with X^3 label of each stroke printed at the bottom, and (d) 3D overlayed representation of all the geometric objects. Note that corner points and multiple points are indicated by spheres on and above the topological curve.

In our first experiment, we collected on-line handwriting data from about 70 students. They were asked to write numerals normally on the computer screen. The result is shown in Table 1. Note that in our description method we do not have to conduct normalization to cope with the size variation of the handwritings.

5 Concluding Remark

Mori and Nishida [7] compared their algebraic approach to handprinted character recognition with the contour analysis technique from the viewpoint of recognition ratio and the number of models, and concluded that the algebraic approach using quasi-topological features is superior to contour analysis technique in error rate and the number of models required.

Our method inherit the same advantages of the al-

Char	Recog.(%)	Rej.(%)	Subst.(%)	Mod.
0	96.3	0.7	0.3	1
1	98.1	1.9	0.0	1
2	95.6	2.4	0.2	3
3	97.7	1.3	0.2	2
4	96.4	3.5	0.1	1
5	97.2	2.8	0.0	3
6	98.3	1.7	0.0	2
7	96.5	1.5	0.2	2
8	94.7	3.3	0.2	1
9	96.6	2.4	0.1	3
	96.7	2.1	0.1	2

Table 1: Results on unconstrained numerals.

gebraic approach, but is significantly different by nature with other methods [2, 8]:

- By decomposing a stroke from pen-down to pen-up into directional line segments and constructing a "single" topological curve from them, we obtain *simple* and *stable* description. Decomposing a curve based on sharp turn [8] or monotone requires more number of models because cusp or monotone is structurally unstable [9]; in other words, a cusp changes to a self crossing and '—' to '/' or '\' under a small perturbation.

- The X^3 label is associated with each curve. The X^3 label provides the minimal and sufficient information to characterize topological turning pattern of both monotonous and non-monotonous curves. If we employ sorely $< ps,\ idr >$ label as in [2], even a simple character like 'Σ' requires three PS-labels $< 3,\ 0 >$, $< 3,\ 3 >$, and $< 3,\ 1 >$ linearly connected together for its description. Note that with X^3 label, 'Σ' is described simply by $< 5,\ 4,\ 2 >$. If we give PS-label $< 0,\ 0 >$ to infinitely cyclic simple closed curve, it is hard to distinguish '0' and '8'. Note that with X^3 label only one model is used to specify '8' in our description, while 10 models were required in PS-label approach [2].

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