

# Automatic Correspondence of Range Images

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## Abstract

*Correspondence is a fundamental part of three dimensional (3D) model building. It is the process of using surface features to determine correspondences between range images. Ideally, we desire a fully automatic system to create 3D models from range data. However, the automatic correspondence part of the model building procedure has proven very difficult. In this paper we present a nomenclature to categorise correspondence methods. A variety of methods are reviewed and classified, including classic examples such as iterative closest point matching and more recent techniques such as bitangent curve matching. A new automatic correspondence algorithm, which combines existing techniques with the novel approach of searching the correspondence probability matrix space, is presented. Finally, results for the algorithm are shown for 2D range profiles.*

## 1 Introduction

Many industrial applications require 3D models of objects and scenes taken from range images. Some examples where these models are used are 3D recognition systems, virtual reality map creation, terrain mapping, industrial model inspection, medical imaging and mining.

The 3D model building process consists of four steps:

1. data acquisition: using a sensing device to obtain sets of range images;
2. correspondence: identifying features in different views which correspond to the same physical surface features;
3. registration: bringing the corresponding features in different views into alignment; and
4. reconstruction: merging the aligned views to build a 3D model.

Ideally, we desire a system that automatically creates 3D models from range images, using the steps outlined above. In this paper, we focus on the automatic correspondence process. The automatic data acquisition, registration and reconstruction problems are more or less solved, yet the problem of correspondence has proven difficult. This paper

addresses automatic correspondence by reviewing existing techniques and by introducing a novel approach to solve the problem.

We first define important correspondence terminology in a novel nomenclature. Key correspondence techniques are reviewed in Section 2, then in Section 3 our novel automatic correspondence algorithm, the Automatic Probability Matrix-based Intrinsic Correspondence Algorithm (APM-based ICA), is proposed. Results from testing 2D range profiles are presented in Section 3.7. We conclude by discussing future work in the area of automatic correspondence.

## 2 Review and classification

In this section we define correspondence and present a nomenclature to distinguish between classes of correspondence techniques. We also present a review of widely used correspondence techniques such as the Iterative Closest Point (ICP) and Random Sample Consensus (RANSAC)-based Data-Aligned Rigidity-Constrained Exhaustive Search (DARCES) algorithms.

### 2.1 The correspondence nomenclature

The different roles of correspondence and registration are often not clearly defined in the literature. Also, correspondence algorithms are not distinctly classified according to the methods used to obtain correspondence between data sets. This section introduces a unifying nomenclature to describe correspondence techniques.

#### Correspondence vs registration

As mentioned, there is often no clear distinction made between the terms *correspondence* and *registration*. Although both are fundamental in 3D model building, they are separate and distinct processes. Surfaces cannot be registered without first determining the correspondences between them.

*Correspondence* is the process of identifying features in different views which correspond to the same surface features. This can be done using a number of different meth-

ods, such as matching signatures from one view onto another [4]. *Registration* is the process of bringing those corresponding features into alignment. Numerous registration methods exist. The most recent ones presented in the literature are multiview techniques that incorporate error modelling for greater alignment accuracy (e.g. see [16, 17]). Some techniques combine correspondence and registration so that correspondences are updated to iteratively determine alignments between views.

In this paper we focus on the process of correspondence. The following sections present a novel terminology to distinguish between different correspondence methods.

### Extrinsic vs intrinsic methods

A number of different correspondence methods have been proposed in the literature. We have noted that each technique falls into one of two categories, which we define as *extrinsic* and *intrinsic*. *Extrinsic* is defined as “relating to the space in which the bearer of the property is embedded” and *intrinsic* as “relating only to the bearer of the property and not to the space in which it is embedded”. These terms appropriately categorise correspondence techniques for the reasons discussed below.

Extrinsic methods depend upon the orientation of surfaces to determine the correspondences between them. These methods are iterative, with the estimated correspondences depending on the existing orientations of the views. They usually require good initial estimates to avoid being stuck in local minima. The ICP algorithm [2] is the classic extrinsic method.

Intrinsic methods are based upon properties of the surfaces themselves, rather than the orientation of those surfaces in space. Unlike extrinsic methods, the intrinsic approaches are generally non-iterative, one-step procedures. Thus, the correspondences yielded by intrinsic methods are a function only of the surfaces, and not of any prior transformation or orientation between these surfaces. An example is the RANSAC-based DARCES method [5].

**Table 1. Key correspondence techniques.**

| EXTRINSIC                           | INTRINSIC                        |
|-------------------------------------|----------------------------------|
| iterative closest point [2]         | geometric histogram matching [1] |
| ICP derivatives (e.g. [13, 15, 19]) | RANSAC-based DARCES [5]          |
| rangefinder calibrations [3]        | graph matching[7]                |
| signature search [4]                | 3-tuple matching [8]             |
| Chen & Medioni [6, 9]               | force functions [10]             |
| mutual information [14]             | spin-images[11]                  |
|                                     | bitangent curve matching [18]    |
|                                     | spherical image matching [12]    |

A variety of correspondence techniques are presented in Table 2.1. These techniques have been categorised as either extrinsic or intrinsic and cover numerous approaches for solving the correspondence problem. The methods are reviewed in the following sections.

## 2.2 Extrinsic methods

The classic extrinsic method, the ICP algorithm [2], was proposed by Besl and McKay in 1992. In this section, we discuss the ICP and its derivatives. We also review other extrinsic methods such as correspondence by maximising mutual information and spherical image matching.

The ICP method [2] determines transformations between two surfaces by considering the Euclidean distance between nearest points, line segments, curves, triangle sets or other shapes defined on the views. The nearest points or shapes are chosen as correspondences, which are used to calculate the required transformations between the views. The matches and alignments are progressively updated in an iterative procedure. The ICP algorithm originated as a solution to the object recognition correspondence problem. Therefore, it only works under the assumption that one surface, of the two surfaces being aligned, is a subset of the other. Several modifications in constraining the *types of matches allowed* have been developed, so as to align mutually partially overlapping surfaces. Proposed solutions include disallowing matches with boundary points [15], limiting the maximum distance between corresponding points [19] and selecting only a certain percentage of best correspondences [13].

Chen and Medioni proposed an algorithm which improves the rate of convergence of the ICP algorithm. Instead of minimising the distance between pairs of matched points, their algorithm minimises the distance between the points on one view and tangent planes on the other [6]. A proposed improvement to this technique investigates the accuracy of the surface normal estimation of noisy data [9]. A minimum variance estimator for computing the transformation parameters was utilised to increase the effectiveness of the algorithm.

Rangarajan *et al.* presented a mutual information algorithm that also follows minimum Euclidean distance point matching. The key difference to the ICP and Chen and Medioni methods is that the weighted distances between each point on one surface and each point of the other are considered as matches [14]. These correspondences define the amount of mutual information between two views in a particular orientation with respect to one another. The algorithm iteratively brings the two surfaces into alignment by maximising the mutual information.

Several extrinsic algorithms that do not apply a minimum Euclidean distance type algorithm have also been proposed. One such technique uses signature search methods. Burtnyk and Greenspan [4] proposed an extrinsic algorithm whereby the pose correction in matching range profiles of two surfaces is iteratively minimised. Reversing rangefinder calibrations [3] to determine correspondences is another unconventional extrinsic method. The first stage of the algorithm determines the locations of points with re-

spect to the rangefinder, so that initial correspondences can be formed between views. The second stage utilises the more conventional extrinsic approach of minimising a Euclidean distance measure between the surfaces.

In the following section, we discuss key intrinsic techniques, which concentrate more on surface properties, rather than surface orientations and transformations with respect to one another.

### 2.3 Intrinsic methods

Unlike most extrinsic methods, intrinsic techniques are generally not iteratively converging. They are often one-step correspondence procedures, which may be repeated a set number of times to find the best matches between views. We review some key intrinsic methods such as 3-tuple and bitangent curve matching.

Geometric histogram matching is a typical intrinsic technique [1]. Pairwise geometric histograms are generated for each triangular mesh facet of the surfaces being matched. First, local correspondences, between individual facets, are hypothesised by matching their histograms. Global correspondences are determined by applying a probabilistic Hough transform to the collection of local hypotheses to find the optimal correspondences between two the surfaces. Guest *et al.* proposed a similar intrinsic technique [10]. Correspondences are hypothesised by matching a selected point from one surface to several points on the other surface, using a Euclidean distance based force function. Then small movements are applied to the match point, and the matches are re-estimated for each perturbation. This creates a distribution of tentative correspondences, from which the optimal match can be calculated.

Direct surface matching techniques have also been presented in the literature. 3-tuple matching [8] is a geometric approach whereby three dispersed seed points (a 3-tuple) are selected on one surface, and the other surface is searched for compatible 3-tuples. Transformations are calculated for each 3-tuple match and a heuristic search is used to select the optimal transformation.

A related technique, the RANSAC-based DARCES algorithm [5], selects a triangle on one surface and attempts to find a compatible triangle on the other surface. Again, the transformations for each compatible triangle are calculated and the best transformation is chosen as the one with the most points within a close predefined distance of one another. RANSAC (random sample consensus), a robust estimation technique, is used to randomly select a primary control point on one view and determine a consensus of the best matches of regions on the other.

Graph matching [7] is another geometric-based technique. A graph is constructed over the vertices of each surface and correspondences are determined by matching the vertices of the graphs. However, this technique is extremely

computationally expensive.

Vanden Wyngaerd *et al.* proposed a signature-based correspondence technique that uses bitangent curves as signatures [18]. This intrinsic technique is automatic, however heavy smoothing is required to generate good bitangent curves, and the level of smoothing is data dependent, making the algorithm application specific.

Another method of matching surfaces without directly comparing the geometric properties of the range data, uses spherical images. Range image vertex curvature data is mapped onto spherical images (under the assumption that the surfaces are homeomorphic to a sphere), and these spherical images are correlated with one another to find the best matches between them. A similar technique uses spin-images [11]. They are created by transforming oriented 3D points onto local 2D bases, and contain information of the entire surface, which ensures a more discriminating matching procedure. The spin-images are compared with one another and the matches are ranked using a correlation coefficient. Highly ranked matches are used as initial correspondences. Final correspondences are established using an ICP-type technique.

## 3 The APM-based ICA

In this section, we present our novel correspondence method, the Automatic Probability Matrix-based Intrinsic Correspondence Algorithm (APM-based ICA). From the review, we noted that the key to developing an automatic correspondence technique is to ensure that it can determine good initial correspondences. Many suitable extrinsic techniques exist, which can bring surfaces into final alignment, yet no widely applicable fully automatic intrinsic correspondence algorithm has been proposed. In this section we propose a novel technique, the APM-based ICA, and show how it relates to the existing literature. Section 3.1 looks at the correspondence problem from the broader view, where we consider a Correspondence Probability Matrix (CPM) space that contains all possible matches between two surfaces. Section 3.2 then considers developing an algorithm that chooses the best match between two surfaces. Sections 3.3 - 3.5 explain the stages of the algorithm in more detail and show how our algorithm actually uses a synergy of ideas from existing correspondence techniques. The APM-based ICA algorithm is presented in Section 3.6 and results from testing the algorithm on 2D range profiles are discussed in Section 3.7.

### 3.1 The CPM space

The correspondences between two surfaces  $X$  and  $Y$  can be defined by a correspondence probability matrix  $\mathbf{P}_c$ . The CPM represents of the probability of a correspondence between every point on  $X$  and every point on  $Y$ :

$$P_{c_{ij}} = P(\text{corr}(X_i, Y_j)) \quad . \quad (1)$$

Assuming a match exists, all possible correspondences between  $X$  and  $Y$  exist within the CPM space. An automatic correspondence algorithm is utilised to search the CPM space, to retrieve the best possible match between two views.

A key step in an automatic correspondence algorithm is to define exactly what is meant by a correspondence between the points on different views. Correspondence matrices are built using these definitions. An ICP for example, uses an exclusive binary process: a point in  $X$  can match with exactly one or zero points in  $Y$ . These matches are placed in the probability matrix  $\mathbf{P}_c$ , with the MI algorithm on the other hand,  $\mathbf{P}_c$  depends on the distances between each point on  $X$  and each point on  $Y$ , and a weighting exponent. An example of the differences is shown in Figure 1. In summary, two important concepts must be understood to develop an automatic correspondence algorithm. Firstly, all possible correspondences between two surfaces  $X$  and  $Y$  can be defined in a CPM space, and secondly, each CPM in the space is built using a unified definition of a correspondence between two points.

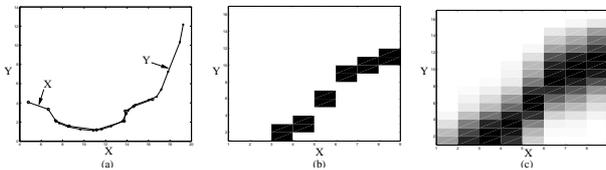


Figure 1. ICP and MI results of partially overlapping curves

### 3.2 Developing an automatic correspondence algorithm

A large number of CPMs make up the CPM space for each surface pair  $X$  and  $Y$ . It would therefore be computationally inefficient to search the entire space for the best CPM. We address this problem by proposing a random procedure whereby CPMs are created for randomly selected regions on  $X$  and  $Y$ . Each  $\mathbf{P}_c$  is then evaluated, and after running the algorithm a set number of times, the best  $\mathbf{P}_c$  is chosen as the best match between the views.

Correspondences are determined by finding regions on surfaces  $X$  and  $Y$  which exhibit the same geometric properties. The first step in a correspondence algorithm is to specify a small control region on one surface  $X$  and determine whether or not it matches a selected region on  $Y$ . Various methods for specifying the small control segment are given in the literature. For example, geometric [5] and signature-based [18] techniques are often employed to characterise these segments. The geometrical properties or the signatures of both the segment on  $X$  and the one on  $Y$  are compared to determine how well the two regions match. If the initial match is significant, a larger region around the

initial control segment can be evaluated. However, before a larger region is evaluated, the control points are used to seed the CPM, so that  $\mathbf{P}_c$  can be used to apply an initial transformation of one surface onto the other.

After the transformation, we allow  $\mathbf{P}_c$  to grow, so that points around the control segment on  $X$  are evaluated against those on  $Y$ . This can be done using a number of properties. For example, probabilities of correspondences can be evaluated by considering the difference in distance between the two surfaces, such as in an ICP or MI algorithm.  $\mathbf{P}_c$  can be built using numerous properties, provided that each selected alignment is evaluated using the same criteria.

After  $\mathbf{P}_c$  has been created, the correspondence needs to be evaluated using a match metric. This quantitative analysis is required to compare a large number of CPMs, so the best one can be selected to determine the best correspondences between  $X$  and  $Y$ . A number of techniques can be used to determine how well two surfaces match. In an ICP-based CPM for example, the number of points on  $X$ , which are within some small preset threshold distance of their closest neighbours on  $Y$ , are counted (e.g. [5]).

The steps described above outline the basic procedure required to develop our automatic correspondence algorithm. This type of algorithm produces good initial correspondences between surfaces by creating numerous CPMs and selecting the one with the best match. The following sections describe the steps of the algorithm in greater detail.

### 3.3 Defining search regions

The first step in our algorithm is to define an initial control region on  $X$  and determine whether or not it matches a selected region on  $Y$ . The segments are chosen using the geometrical properties in a small region of both views. This step in our algorithm is inspired by the DARCES technique which selects three points, forming a triangle, on view  $X$ , and then searches  $Y$  to find a triangle with similar geometric properties.

In our algorithm, we choose a control point  $x_{P1}$  randomly on  $X$ , and create a triangle using two other points,  $x_{P2}$  and  $x_{P3}$ , in the region. We then randomly select a point  $y_{P1}$  on  $Y$  and determine whether or not a matching triangle can be found. The matching triangle is determined by searching the region around  $y_{P1}$  using the geometric properties of the triangle formed by  $x_{P1}$ ,  $x_{P2}$  and  $x_{P3}$  (refer to [5] for a more detailed explanation). If a suitable triangle is found, the algorithm proceeds to further evaluate the match between the two regions.

### 3.4 Building the CPM

If two suitably matching triangles have been found on  $X$  and  $Y$ , the CPM  $\mathbf{P}_c$  is created. The initial control points seed the CPM. The CPM is then used to estimate a transformation between  $X$  and  $Y$ . Once the transformation has

been applied,  $\mathbf{P}_c$  is 'filled' by considering the probabilities of correspondences between every point on  $X$  and every point on the transformed surface  $Y$ .

The probabilities of correspondences are formed using the following definitions. The first probability assignment is based on differences in distances [14], which is an effective method for forming  $\mathbf{P}_c$ . Each element in  $\mathbf{P}_c$  is given by:

$$P_{c_{ij}}(\mathbf{f}) = \frac{\exp(-\alpha D_{ij}^2(\mathbf{f}))}{\sum_{ij} \exp(-\alpha D_{ij}^2(\mathbf{f}))}, \quad (2)$$

where  $\alpha$  is the weighting exponent, and  $D_{ij}^2$  is a matrix representing the squared Euclidean distance between each point  $x_i$  on  $X$  and each point  $y_j$  on  $Y$ . This is used to initialise the entire  $\mathbf{P}_c$ . Additional constraints are then placed on the CPM to eliminate as many false matches as possible. We address this issue by using ICP-based constraints discussed in Section 2.2 (e.g., [13, 15]). Once these constraints have been applied,  $\mathbf{P}_c$  is complete.

### 3.5 The CPM match metric

It is necessary to evaluate the CPM after each repetition of the algorithm so that it can be compared with other  $\mathbf{P}_c$  matrices. The evaluation must indicate how well two surfaces  $X$  and  $Y$  correspond under a given CPM. A current correspondence technique has been utilised to demonstrate how mutual information is maximised for the best correspondences between two views [14]. Hence, we utilise MI to rank  $\mathbf{P}_c$  in our automatic correspondence algorithm.

The equation for MI is given by:

$$MI(\mathbf{P}_c) = \sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} P_{c_{ij}} \log \frac{P_{c_{ij}}}{\sum_{n=1}^{N_X} P_{c_{nj}} \sum_{l=1}^{N_Y} P_{c_{li}}}, \quad (3)$$

where  $\sum_{n=1}^{N_X} P_{c_{nj}}$  and  $\sum_{l=1}^{N_Y} P_{c_{li}}$  are the marginal probabilities, and  $N_X$  and  $N_Y$  are the number of points in  $X$  and  $Y$  respectively. Equation 3 is maximised when optimal correspondences between the two views are formed. Based on this principle, we note that MI is a suitable match metric for evaluating each CPM in every repetition of our algorithm.

### 3.6 The algorithm

Algorithm 1 presents details of our proposed method, based on the ideas discussed in Sections 3.3 - 3.5. The intrinsic method is utilised to automatically obtain the best correspondences between two partially overlapping surfaces.

Algorithm 1 is completely independent of initial estimates. The search procedure is non-iterative and can therefore be run in a parallel fashion. Results of the APM-based ICA are presented next.

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#### Algorithm 1 The APM-based ICA

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1. Randomly select a point on the view  $X$  and one on the view  $Y$
  2. Apply a DARCES type algorithm to match the points
  3. If the points match successfully, apply the following procedure:
    - Define a probability matrix  $\mathbf{P}_c$  using the control points
    - Transform  $Y$  onto  $X$  using  $\mathbf{P}_c$
    - Apply MI constraints to consider the matches of all points on  $X$  and all points on  $Y$
    - Apply additional ICP-based constraints to further refine the probability matrix
    - Rank  $\mathbf{P}_c$  using Mutual Information
- Select the best correspondences between  $X$  and  $Y$  by choosing the  $\mathbf{P}_c$  yielding the highest MI.
- 

### 3.7 Results

The algorithm proposed in the previous section can be applied to a variety of range profiles. A variety of 2D range data is utilised to test the capabilities of the APM-based ICA. The range data consists of partially overlapping 2D curves. During the preliminary tests, percentage of overlap, number of repetitions of the code and the mutual information are utilised to analyse the algorithm. We also test the robustness of the algorithm by using relatively noisy input data. The results of our automatic correspondence technique are shown in Figure 2.

The first dataset ( $X_1, Y_1$ ), shown in Figure 2, consists of two curves with approximately 75% overlap. These overlapping segments are brought into almost perfect correspondence within three thousand repetitions (of correspondence matches in the algorithm). The algorithm responds well to this input because of the large percentage of overlap, and the good variation in curvature of the profile. The MI of this match is 2.24.

The second dataset ( $X_2, Y_2$ ) required that a greater number of correspondence matches are evaluated to provide good matches between the surfaces. Ten thousand repetitions were performed to ensure good correspondences were found between views  $X_2$  and  $Y_2$ . Although the curvature of the profiles varies significantly, the amount of overlap between the two segments is only about 50%. This reduces the probability of finding two initial matching regions, hence the need for more repetitions. The MI for this match is 2.34.

The third dataset ( $X_3, Y_3$ ) was incorrectly matched by our algorithm. This is due to a number of important reasons. Firstly, profile  $Y_3$  only contains one small distinctive feature which is highly embedded in noise. Secondly, the amount of overlap between the two segments is only 25%, and there are no distinctive features in the overlapping regions. Even after ten thousand correspondence matches, the maximum MI (2.80 in this case) could not match the noisy, featureless  $Y_3$  profile in its correct place. It must be noted

that no algorithm could match  $X_3$  and  $Y_3$  due to these noise levels and the non-distinctive features on  $Y_3$ .

From the results discussed, it can be seen that mutual information is a good match metric for a variety of data. In future, we will consider determining the number of correspondence matches required to produce a high MI.

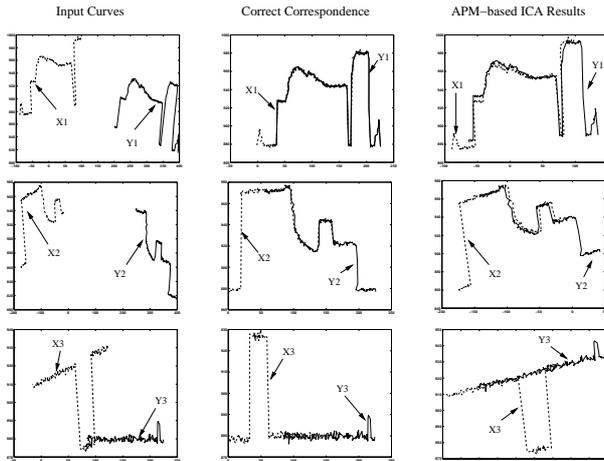


Figure 2. APM-based ICA results.

#### 4 Conclusions and future work

In this paper, we discussed the problem of automatic correspondence for range images and developed a nomenclature to distinguish between correspondence techniques. A variety of extrinsic techniques (e.g., ICP matching) and intrinsic techniques (e.g., RANSAC-based DARCES method) were reviewed. We presented a novel automatic correspondence algorithm, the APM-based ICA, which is a synergy of existing methods and searches the CPM space to find the best correspondences between two surfaces. Finally, we showed results using 2D range profiles as inputs to our algorithm.

The initial results were very promising. Future work will help to refine our correspondence technique to develop a fully automatic match method, which can be applied to a wide variety of 3D modelling applications. We will improve the algorithm by statistically determining the number of repetitions of the code required for any given data sets to achieve good correspondences between surfaces, examining the behaviour of MI with respect to surface properties such as sampling density and scale, extending the algorithm to 3D, and also generalising the algorithm to incorporate multiple views.

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