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Peak-to-Average Power Reduction in Digital Television Transmitters

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is one of the modulation techniques used in digital video broadcasting systems. A significant disadvantage of OFDM is that it has a high peak-to-average power ratio (PAPR). This means that the output amplifiers of digital television transmitters must be designed to handle signals that are far in excess of the mean levels. These amplifiers are expensive. One way to reduce the PAPR is to digitally limit the peak value before the signals reach the amplifier. Various clipping techniques have been described in the literature. This paper describes the application of the most effective of these techniques to the European digital video broadcasting system. It is shown that the PAPR can be reduced without significant increase in bit error rate. In practical systems, amplifier non-linearities cause unwanted out-of-band power which may interfere with adjacent television channels. Simulation results are presented for systems with and without clipping and for varying amplifier characteristics showing that clipping substantially reduces the level of out-of-band power.

1. Introduction

Digital television broadcasting is currently being introduced throughout the world. Three different transmission standards have been developed. Two of these use orthogonal frequency division multiplexing (OFDM) as the modulation technique. The third, which is used in the US, is based on vestigial sideband modulation (VSB). There has been heated debate about the relative merits of VSB and OFDM based systems. On most measures OFDM outperforms VSB, however OFDM has the significant drawback of having a high peak-to-average power ratio (PAPR). This means that the output amplifiers in the television transmitters must be designed to handle peaks that are far in excess of the mean power. These amplifiers are expensive to construct and maintain. The amplifiers must also be very linear over a large dynamic range. Any non-linearity results in intermodulation and out-of-band power that will interfere with adjacent television channels. This paper shows how a recently developed PAPR reduction technique can be applied to OFDM in digital television.

2. Background on OFDM

OFDM is a multicarrier system. This means that the available bandwidth (6MHz, 7MHz or 8MHz for digital television depending on the country) is divided into many narrow bands. The data is transmitted as a large number of lower bit rate streams on these bands. The European digital video broadcasting (DVB) standard specifies two modes, the 2K mode with approximately 2000 narrow bands and the 8K mode with approximately 8000 bands [1]. OFDM transmission and reception is possible because the modulation and demodulation of these many subcarriers can be achieved using Fast Fourier Transforms (FFT). The FFT is an efficient algorithm for calculating the discrete Fourier Transform (DFT).

Figure 1 shows the block diagram of an OFDM system. The data to be transmitted is divided into lower bit rate streams and mapped onto the available subcarriers using quadrature amplitude modulation. In DVB 4QAM, 16QAM and 64QAM modes can be used depending on the overall data rate required. The resulting complex data stream is divided into vectors of length *N*. $a_{0,i} \cdots a_{N-1,i}$ is the *i*-th *N* point vector. An *N*-point inverse DFT (IDFT) of the vector $a_{0,i} \cdots a_{N-1,i}$ results in modulation and multiplexing of all of the subcarriers. $b_{0,i} \cdots b_{N-1,i}$ is the *i*-th vector of samples of the complex baseband signal. These samples must be converted to serial form, filtered and/or interpolated, converted to analogue and modulated onto a high frequency carrier at frequency f_c before transmission.

The precise order and form of these operations can vary in practical implementations. Figure 1 shows all *N* subcarriers being used, but in most practical OFDM systems the band-edge subcarriers are not used. For example in the DVB 2K mode only 1705 subcarriers carry data although a 2048 point FFT is used. In the receiver a forward DFT is used to demodulate and demultiplex the signal.

3. PAPR Reduction Techniques

Although the PAPR for OFDM is very large, high magnitude peaks occur relatively rarely and most of the transmitted power is concentrated in signals of low

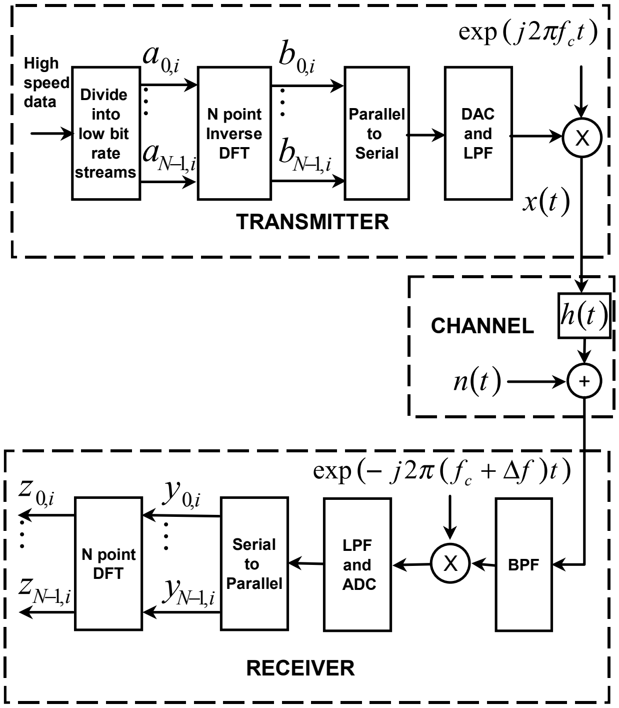


Fig. 1 Block diagram of an OFDM system

amplitude. For DVB the distributions of the real and imaginary components of the baseband signals are very close to Gaussian. This means that the amplitude has a Rayleigh distribution.

The simplest approach to reducing the PAPR of OFDM signals is to clip the high amplitude peaks. Various clipping techniques have been described in the literature [2-3]. Some limit the peak signals by clipping $b_{0,i} \cdots b_{N-1,i}$, the outputs of the IDFT, before filtering or interpolation. However the signal must be filtered or interpolated before transmission and this will cause peak regrowth, so clipping before interpolation is not very effective in reducing the PAPR [4].

To avoid the problem of peak regrowth, the signal can be clipped after interpolation. However this causes very significant out-of-band power. Some papers have described clipping of the interpolated signal followed by filtering [2]. The filters used were complicated and significantly distorted the wanted signal. Filtering also causes peak regrowth, although this is less than for clipping before interpolation. A recent paper [5] describes a new PAPR reduction technique in which clipping of the interpolated signal is followed by a different form of filter.

Figure 2 shows a block diagram of this PAPR reduction scheme. The input vector $a_0 \cdots a_{N-1}$ is first converted from the frequency domain using an oversize IDFT. For an oversampling factor of I_1, the input vector is extended by adding $N(I_1-1)$ zeros in the middle of the vector. This results in trigonometric interpolation of the time domain signal [6]. The interpolated signal is then clipped. In this paper, hard-limiting is applied to the amplitude of the complex values. However the technique is quite general and any other form of non-linearity can be applied. The clipping ratio, *CR,* is defined as the ratio of the clipping level to the root mean power of the unclipped baseband signal. The clipping is followed by frequency domain filtering to reduce out-of-band power.

The filter consists of two DFT operations. The forward DFT transforms the clipped signal back into the discrete frequency domain. The in-band discrete frequency components of the clipped signal $c_0 \cdots c_{N/2-1}, c_{NI_1-N/2+1} \cdots c_{NI_1-1}$ are passed unchanged to the inputs of the second IDFT while the out-of-band components, $c_{N/2+1} \cdots c_{NI_1-N/2}$ are nulled. In systems like DVB where some band-edge subcarriers are unused, some of the inputs $a_{0,i} \cdots a_{N-1,i}$ are zero. Because of the properties of the DFT, it is the middle values of the vector $a_{0,i} \cdots a_{N-1,i}$ which correspond to the highest frequency

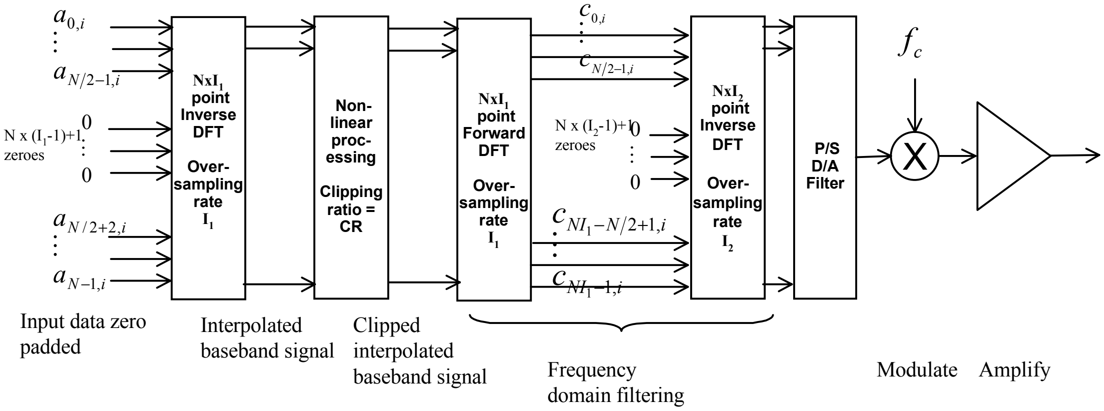


Figure 2. Block diagram of new peak reduction technique

baseband components and thus to the band-edge components after conversion to the carrier frequency. In this case, in the filtering process these components are also nulled.

The second IDFT is followed by serial/parallel conversion, digital to analogue conversion and modulation. This simple filter structure is effective because the wanted signal is an OFDM signal, which is the sum of discrete frequency components in each symbol period. The filter must therefore have as little effect as possible on the in-band *discrete* frequency domain while attenuating as much as possible any out-of-band components. Because the filter operates on a symbol by symbol basis there is no filtering across symbol boundaries and so no resultant intersymbol interference. The filtering does cause some peak regrowth however this is much less than in other clip and filter schemes [5].

4. Peak-to-Average Reduction in DVB

There are a number of important measures of performance for a peak reduction technique. The most basic is the effectiveness of the technique in reducing the PAPR. Figure 3 shows the complementary cumulative distribution function for the case of the DVB 2K mode, with $N = 2048$, 343 unused subcarriers and $CR = 6\text{dB}$. Two cases are shown: $I_1 = 1$, that is no oversampling and a 2048 point IDFT; and $I_1 = 2$, that is an oversampling factor of two and a 4096 point IDFT. In both cases, after clipping and filtering, a proportion of the signal samples exceed the clipping level. This is because the filtering causes peak regrowth. However peak regrowth is significantly less for $I_1 = 2$. Note the logarithmic scales. Simulations for the 8K mode case (not shown) give similar results.

Figure 4 shows the distribution of instantaneous signal power as a function of *CR* for the DVB 2K mode parameters. The graphs show the power that a given percentage of signal samples is below. In each case the levels are normalized to the mean power of the signal after processing. Results are shown for $I_1 = 1$, $I_1 = 2$ and for OFDM with no clipping. Clipping after interpolation is more effective than clipping before interpolation in reducing the dynamic range of the signal. This is particularly the case for $CR < 6\text{dB}$. Increasing the oversampling rate I_1 further does not give significant improvement in performance.

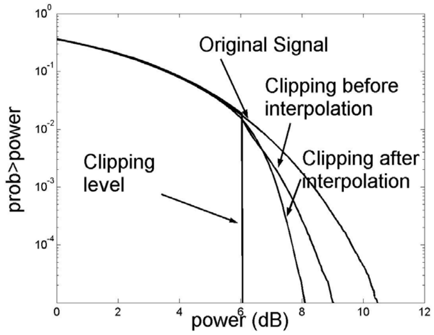


Figure 3 Cumulative distribution of signal amplitudes when not all subcarriers are used. Parameters for DVB '2k' mode, $N = 2048$, 343 subcarriers unused.

5. In-band Distortion caused by clipping

Clipping the OFDM signal results in distortion to the wanted signal. This will cause some increase in bit error rate (BER). It has been have shown that clipping at the transmitter has two effects on the transmitted signal constellation [7]. There is an overall shrinking of the constellation as the average transmitted signal power is reduced and there is a noise like effect that is caused by the distortion. This is called clipping noise. The shrinking of the constellation can be corrected by adjusting the amplifier gain so that the average transmitted power is the same as a system without clipping. Thus, for a given average transmitted power, only the clipping noise, not the shrinking, affects the overall BER performance. Another factor that makes the overall degradation less than might be expected is that the clipping noise is added at the transmitter not at the receiver. Therefore in a fading channel the clipping noise fades along with the signal. This has less effect than noise added at the receiver, which is not subject to fading. The precise interaction of this effect with the error-correcting coding used in all practical OFDM systems is an area of on-going research.

In a practical system the signal must be amplified before transmission. This amplifier will not have an infinite dynamic range so some additional clipping will occur here. Figure 5 shows the signal-to-clipping-noise

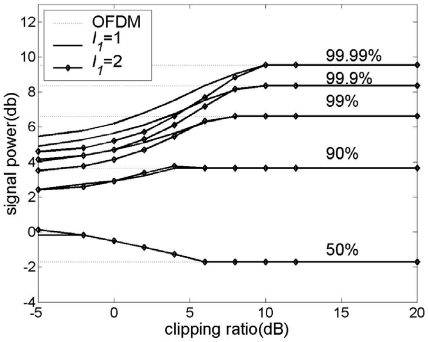


Figure 4. Distribution of instantaneous signal power for clipping before and after interpolation and undistorted OFDM when not all subcarriers are used. Parameters for DVB '2k' mode, $N = 2048$, 343 subcarriers unused.

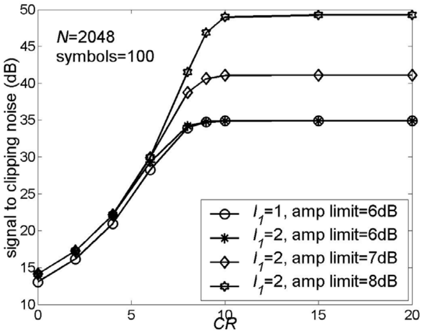


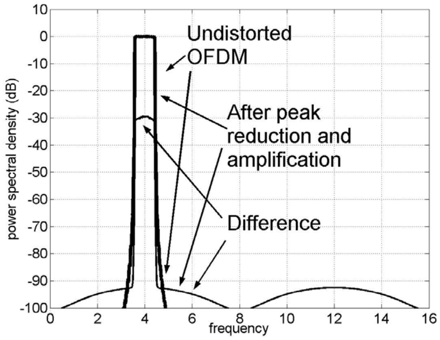
Figure 5. Signal to clipping noise ratio as a function of *CR*.

ratio (SCNR) as a function of CR. These graphs include the effect of non ideal amplification. Results are shown for amplifiers that limit at 6dB, 7db and 8dB above the mean power of the unclipped signal. DVB transmitter amplifiers are typically operated with a 7dB input back-off for systems with predistorters which correct for amplifier non-linearities and 10dB for systems without any linearising techniques. Figure 5 demonstrates that even extreme clipping causes only moderate values of SCNR. This is because the main effect of clipping is to shrink the entire constellation rather than to add clipping noise. The extra clipping noise added by the amplifier is negligible if the amplifier limits more than 2dB above *CR*.

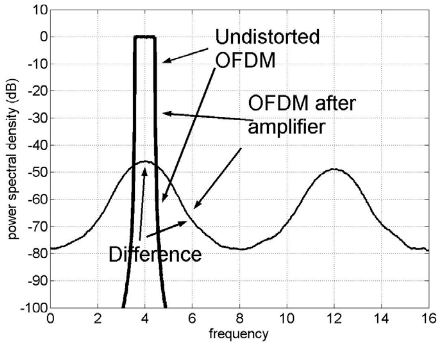
6. Out-of-band power caused by non-ideal amplification

The main reason for using PAPR reduction is to minimize the out-of-band power of the transmitter. This PAPR reduction scheme by itself causes no added out-of-band power however when the signal is amplified by a non ideal amplifier some out-of-band power will result. The level of this depends both on the dynamic range of the amplifier and how linear the amplifier is within its operating range.

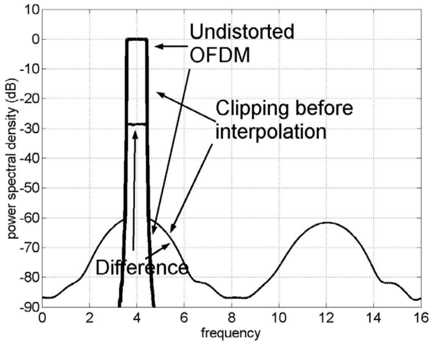
Figure 6(a) shows the spectrum that results after modulation and amplification with an amplifier with limited dynamic range. The simulations are for $CR = 6$ dB and $I_1 = 2$. The clipped and filtered baseband signal then modulates a carrier of frequency f_c. In these simulations f_c is four times the nominal bandwidth but in practice the carrier frequency would be much higher. The modulated signal is then amplified. The amplifier limits for a baseband signal 7dB above the root mean baseband power but is perfectly linear up to this level. For comparison the spectra for OFDM with no peak reduction (Fig. 6 (b)) and OFDM with clipping before interpolation with nulling of band edge subcarriers (Fig. 6(c)) are also shown. The out-of-band spectra are characterized by 'shoulders' close to the wanted band and peaks around harmonics of the carrier signal. It is easy to filter out the harmonics but more difficult to reduce the level of the shoulders. For the case shown in Fig. 6 the shoulders using the new technique are approximately 95dB down compared with 60dB for clipping before interpolation and 45dB for no peak reduction before amplification.



(a) Clipping after interpolation ($I_1 = 2$)



(b) No peak reduction



(c) Clipping before interpolation ($I_1 = 1$)

Figure 6. Spectra when signals are amplified with a perfect limiting amplifier

Amplifiers are never perfectly linear. The effectiveness of peak reduction techniques depends strongly on the precise characteristic of the amplifier [8]. The following simulations use the Rapp amplifier model [8] in which the output value is related to the input value by

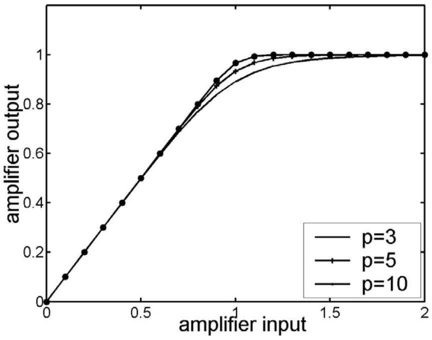


Figure 7. Input/output characteristic of Rapp amplifier

$$y = \frac{x y_{\max}}{\left(1 + x^{2p}\right)^{\frac{1}{2p}}} \tag{1}$$

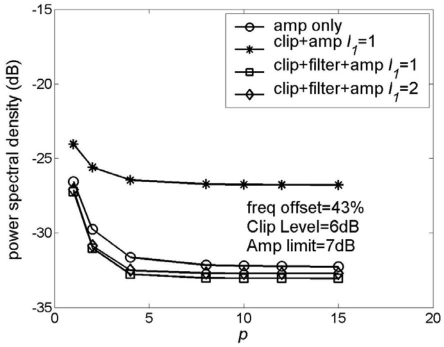
As p increases the amplifier becomes more linear. Figure 7 shows the characteristics of a Rapp model amplifier.

When this model of amplifier is used, the resulting spectra are similar to the one in Fig. 6 but the level of the shoulders depends strongly on *CR* and *p*. Figure 8 illustrates this dependency.

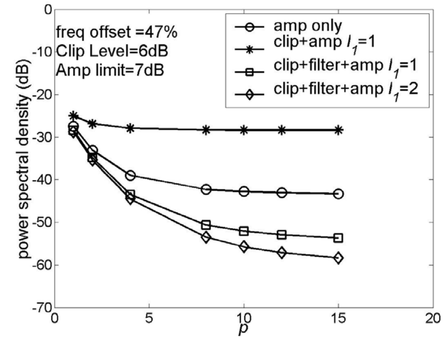
Figures 8 (a) – (c) show the power spectral density (PSD) at normalized frequency offsets of 43%, 47% and 51% from the carrier frequency as a function of p for $CR = 6\text{dB}$ and $y_{\max} = 7\text{dB}$ above the rms. value of the unclipped signal. The frequency offset is normalized to the 'nominal' signal bandwidth. This is the bandwidth the signal would have if all the available 2048 subcarriers were used. The offsets are chosen to be the first three breakpoints on the DVB 2K spectral mask. Four cases are shown: amplification with no peak limiting, clipping before interpolation with no nulling of band edge subcarriers, clipping before interpolation with nulling of the band edge subcarriers and clipping after interpolation with nulling of out-of-band subcarriers. For frequency offsets of 43% and 47% clipping before interpolation with no nulling of subcarriers gives a high PSD which is relatively independent of p and $y_{\max}$. This is because this frequency is still within the nominal bandwidth of the signal and the clipping noise is at the 'in-band' level.

For a frequency offset of 43% there is little variation between the other three cases or with $y_{\max}$. This is because the PSD at this point is still dominated by the roll-off of the wanted signal. The DVB 2K mode specifies a PSD level of –32.8dB at this frequency.

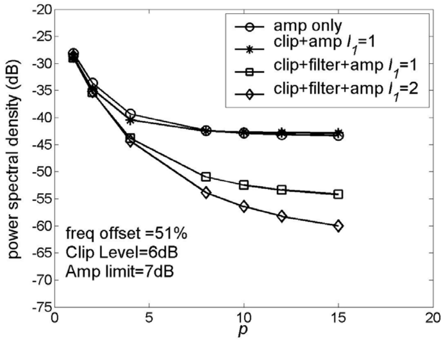
For frequency offsets of 47% and 51% the two versions of the new technique have substantially lower PSDs than simple clipping, or than the system with no peak limitation. The version with interpolation before clipping ($I_1 = 2$) gives a lower PSD than clipping before interpolation ($I_1 = 1$). The improvements gained by the new techniques increase with increasing p and increasing difference between *CR* and $y_{\max}$. The DVB standard specifies a PSD of –57dB at 51% offset. This would be met by the new techniques for $p \approx 7$ and $y_{\max} = 7.5\text{dB}$, whereas without peak limitation the PSD would be -45dB and further filtering would be required.



(a) Frequency offset 43%



(b) Frequency offset 47%



(c) Frequency offset 51%

Fig. 8 Power spectral density for $CR = 6\text{dB}$ and $y_{\max} = 7\text{dB}$ as a function of amplifier parameter, *p*.

7. Conclusions

The application of PAPR reduction scheme to the European DVB system has been described. It is shown that significant PAPR reduction can be achieved without any increase in out-of-band power. Some in-band distortion results but this will have negligible effect on the overall BER in most systems. If necessary particularly critical subcarriers such as pilot tones can be corrected during the filtering process so that these undergo no impairment.

Simulations are presented for the performance of the system when non-ideal amplifiers are used. It is shown that the level of out-of-band power depends on both the degree of PAPR reduction and on the linearity of the amplifier. It is shown that lower amplifier back-offs can be used without creating unacceptable out-of-band noise. The PAPR reduction technique allows simple trade-offs between in-band distortion, amplifier back-off and amplifier linearity. The technique is completely compatible with other aspects of transmitter design such as windowing and filtering and requires no changes in the receiver so can be adopted without any change to telecommunications standards.

8. Appendix – Simulation Details

The power spectral density of an OFDM signal depends very strongly on the precise nature of transitions at symbol boundaries. Abrupt transitions result in a slow spectral roll-off. In many OFDM systems some form of windowing or filtering is used to improve the spectrum. This paper is concerned with the out-of-band spectra due to PAPR and amplifier non-linearities so windowing was applied to smooth the transitions and increase the roll-off so that changes in out-of-band power due to clipping could be more clearly identified. The windowing technique was the one described in [8, Chapter 2]. A cyclic prefix and postfix of length $16T/N$ where T is the original symbol period was added to each symbol. The resulting extended symbol was windowed using a Nyquist window, with roll-off chosen so that all of the prefix and postfix were windowed but the original symbol was unchanged. Adjacent symbols were then overlapped so that the windowed cyclic prefix of one symbol was added to the windowed cyclic postfix of the preceding symbol. SCNR calculations were made using only the original unwindowed section of the symbol, and so were unaffected by the process.

The power spectral density was calculated using the MATLAB PSD function, which is based on the Welch periodigram method. Signal sections of 8192 samples were used and sections were overlapped by 4096 samples. A Hanning window was used.

When a signal is modulated onto a carrier the ratio of peak to rms. voltage is increased by 3dB. Amplifier back-offs are usually specified taking this into account. In this paper a similar convention was used in specifying the amplifier limit $y_{\max}$. For example $y_{\max} = 7\text{dB}$ meant that the amplifier limited when the baseband signal was 7dB above the rms. value. This means that it limited for a modulated signal that is 10dB above the rms. power of the modulated signal.

9. References

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