

## 3D Shape Recovery Based on Radon Transform

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### Abstraction

*The focus of this paper is to present a new approach for recovering 3D shape of an object. We exploited the idea of image reconstruction from projection to construct the cross-section image of the object. Each of the cross-section images is computed from a series of photographs taken at a number of angles around the object. A corresponding intensity profile (row) on each digitized photograph resembles a projection data, which can be used to reconstruct a cross-section image of the object. To extract a 2D coordinate of sectional contour of the object, edge detection is performed followed by locating the x-y position of the pixel constituting the detected edge. Stacking of 2D contours yields a 3D coordinate of the object. To provide better 3D visualization, we also performed a surface-rendering technique on the volumetric data constructed from a stack of cross-section image. Unlike conventional 3D-shape recovery method which uses stereo pairs of camera, our technique does not suffer from a laborious corresponding problem. The 3D – acquisition system is simple, as it requires only a digital camera and a rotating platform. Our purposed method is tested to perform 3D-shape recovery of a variety of objects. The result is very promising.*

### 1. Introduction

3D shape recovering and modeling is the first step of the measurement chain in a number of applications including robot navigation, machine inspection, 3D object recognition, distance measurement, level control, profilometry, displacement measurement, and so on. Many techniques have been purposed for 3D shape recovering including stereo disparity [1-2], laser range finder [3-5], structured light [6-14], shape from shading [15-19], optical flow [20-24], etc.

Stereo disparity is the straightest forward for 3D modeling which uses a triangulation procedure. This method requires the knowledge of global position,

orientation of each camera and model of the camera. It also has to solve correspondence problem between left and right image, which is computationally expensive. Projection of structured light, either coherent or incoherent, allows an easy-solution of the correspondence problem. This method uses a special designed light source to project sheet or beam of light with a known *a priori* spatial distribution onto the scene casting lines or points (dots) on the objects. A video camera is used to visualize the structured light projection (lines or points) on object surface from angles. The major advantage of this technique is that it brings out the natural discontinuity/monotony properties of the object surfaces by replacing them with the artificial features of structured light projection which is more readily recovered and interpreted by computer vision. In structured-light 3D-shape recovering method, corresponding process must match each projected point (dot or intersection of grid) on the structured-lighting image with its producer on grating. Many algorithms have been developed to solve this problem including time modulation [12], spatial encoding, [13] and color coding [14]. Laser range finder gets range information either directly through time-of-flight measurement or indirectly with the triangulation technique. High accuracy can be obtained from this method. Similar to 3D surface point construction using point light projector and/or single slit light projector, this method also involves in some sort of mechanical scanning which is often slow and fragile. Recovering a 3D shape from shading exploits the photometric reflection properties of objects. An image of a smooth object with uniform surface reflectance properties exhibits smooth variations in the intensity of the reflected light referred to as *shading*. This information can be used to determine the shape of the object. The goal is to compute, in each point of the picture, the orientation of the normal vector which can be integrated to give local surface shape [15]. Recovery of shape from shading is very difficult computationally. Optical flow has been widely used for 3D modeling or recovering 3D structure from motion. Optical flow is the resulting apparent motion in the image that is caused by the camera (human eye) moves in the 3D

scene. The optical flow constraint equation describes the direction and the speed of motion of the features in the image and relates the 3D structure of the world to the 2D measurements. The optical flow based method is quite sensitive to noise and cannot capture rapid motion. Optical flow requires the computation of second derivative and hence computational expensive.

In this paper, we propose a novel method for 3D-shape recovery by exploiting the idea of image reconstruction from projection. In this technique, a series of photographs is taken at a number of angles around the object. Each intensity profile (row) of the digitized photograph taken from different angle resembles a projection or radon-transformed data, which is used for reconstructing a cross-section image. To obtain the x-y coordinate, edge detection is performed on the reconstructed cross-section image. 2D coordinate of the edge of all rows is then stacked to obtain a 3D coordinate. Surface rendering of the object is also performed on a volumetric data constructed from a stack of the reconstructed cross-section images. The advantage of the proposed technique is that it requires only a digital camera, or normal camera with a scanner, requires no mechanical scanning and a laborious correspondence problem.

This paper is organized as follows. Section 2 discusses theory involved in image reconstruction from projection. Section 3 explains implementations and results of the proposed method. Discussions and conclusions are given in section 4.

## 2. Tomographic Image Reconstruction from Projection.

An important problem in image processing is to construct a cross section of an object from several images of its trans-axial projection- a so-called tomographic process. A projection  $p(s)$  is a shadowgram obtained by illuminating an object with an electromagnetic source, such as x-ray in CT,  $\gamma$ -ray in PET and SPECT, etc. By rotating the source-detector assembly around the object, projection views from different angle  $p_\theta(s)$  can be obtained. Mathematically,  $p_\theta(s)$  is equivalent to the radon transform  $g(s, \theta)$  [25] of the image (figure 2.1)

$$g(s, \theta) = \iint_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - s) dx dy \quad (2.1)$$

The goal of image reconstruction is to obtain an image  $f(x, y)$  of a cross section of the object from these projections. There are two groups of tomographic image reconstruction algorithms. Series expansion method which use iteration [26] such as ART [27] and ISRT [28] can correct for anisotropy [29], ray bending, and non standard sampling geometry but are computational inefficient. Transform method [30] such as Fourier [31] and filtered backprojection [32-33], use Fourier analysis

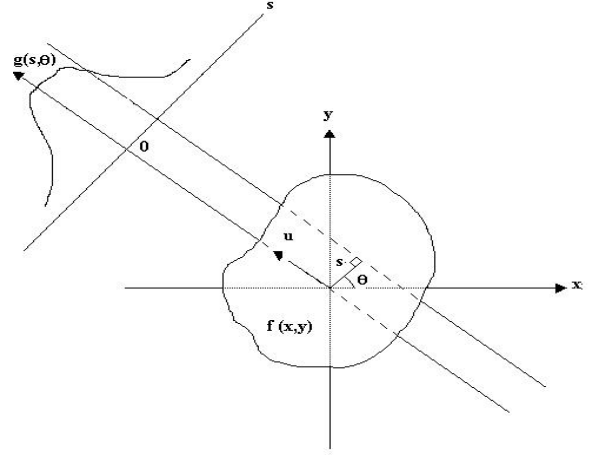


Fig 2.1 Projection Imaging Geometry

which are fast and efficient but requires precise sampling geometry. In Fourier reconstruction method, the cross-section image is obtained by filling the two-dimensional Fourier space with one-dimensional Fourier transforms of the projections in polar-coordinate system and then takes the two-dimensional inverse Fourier transforms. Unfortunately, to perform the inverse two-dimensional Fourier transform, Fourier value on all the raster grids is required. An interpolation from polar to raster grid is needed. This interpolation is prone to aliasing effect.

In this paper we use the popular filtered backprojection for image reconstruction from projection. The concept of filtered backprojection is an extension of the simple 'layergram' [34]. With simple layergram, the profile data are aligned at the appropriate angle and backprojected across the computer's two-dimensional representation of the object field. When all the data have been backprojected, an image of the original object is obtained. This crude method produces star artifacts in the image which is caused by a polar coordinated system is being used to produce data [35], which has the effect of underweighting the higher frequency components. To lessen the effect, the convolution backprojection is introduced. The goal of convoluted backprojection is to choose a convolution kernel [25] that eliminates the blurring and leaves the true object image. This is done by introducing leading and trailing negative tails to the transmission data. Backprojecting the filtered profiles then cancel out the undesired components in the final image. Convolution backprojection can be performed in frequency domain - the so-called filtered backprojection-by multiplying the frequency response  $|\xi|$  of the one-dimensional filter with the Fourier transform of the projection before backprojecting the inverse transform of the result, that is [25]

$$f(x, y) = \mathbf{BF}^{-1}\{|\xi| \cdot \mathbf{F}[g(s, \theta)]\} \quad (2.2)$$

where the backprojection  $\mathbf{B} = \int_0^\pi g(x \cos \theta + y \sin \theta)$

$d\theta$ ,  $F$  is the one dimensional Fourier transform and  $F^{-1}$  is the one-dimensional inverse Fourier transform. As in the Fourier method, interpolation is needed in the image plan, but this is now done after calculating the convolutions of each projection. The interpolation is one-dimensional.

In this paper, we exploit the concept of tomographic process to construct a cross-section of the object by taking a series of photographs at a number of angles around the object. In this case, light reflected from the object to the camera acts as a source of a tomographic process. Using light source is reasonable as we are interested only the contour of the object not the interior of the object. To emphasize the object contour, the selected object, which has a bright color, is placed on a dark background. The gray level profile (row) of the digitized image represents the projection data. A series of the corresponding profile of the image taken at a number of angles around the object can be used to construct the cross-section image of the object.

### 3. Implementations and Results

Figure 3.1 shows the configuration of the 3D- acquisition system which consists of a rotating platform and a digital camera. To acquire the 3D information, we placed the tested object on the rotating platform. The platform is capable of rotating with a resolution of 1 degree per step; hence the maximum number of projection per one image reconstruction is 179. The black sheet of cloth or paper is placed behind the object yielding the dark background in the image. A photograph is taken using the digital camera. To minimize the perspective effect, the focal length of the camera used is relatively high. The resolution of the digital camera is 640×480. After each exposure, the platform is rotated once at a setting angle. Using the first intensity profile (row) of every digitized image, cross-section of the first slice is reconstructed



Figure 3.1 Configuration of the System

using the filtered back-projection technique. Ram-Lak filter [25] is considered as the choice of the filter because of its superior performance to construct the image in the absence of noise. The choice for one-dimensional interpolation is linear [25]. The reconstruction is repeated for all of the 480 rows. The resolution of the newly reconstructed image is 256×256. Note that as we are interested only on the contour or the cross section, any gray-level value on the profile would result in the same contour. In practical, it is preferable to adjust the contrast and brightness such the object totally appears in white on the dark background. To obtain the x-y coordinate, we performed an edge detection to extract the sectional contour of the object. The x-y coordinate is then simply determined by locating the x-y position of the detected edge. Examples of two cross-section image and the edge-detected image are shown in figure 3.2. The x-y coordinate of the sectional contours is stacked to obtain a z coordinate and hence a 3D coordinate of the object. To provide the better 3D visualization, we also perform surface-rendering technique on the volumetric data obtained from a stack of 2D reconstructed image. Surface-rendering technique is performed by extracting the iso-surface using the marching cube algorithm [36]. The extracted iso-surface is then illuminated and shaded using widely used Phong [37] and Gouraud [38] shading or illumination model. Figure 3.3 b) shows the 3D coordinate of the mannequin. Figure 3.3. c) shows surface-rendered mannequin. The number of projection used for the mannequin is 60.

The purposed algorithm has also been tested on a variety of objects. Figure 3.4 b) and c) show 3D coordinate and the surface rendering of a coffee cup respectively. The number of projection used for the coffee cup is 72. Figure 3.5 b) and c) show the 3D coordinate and the surface rendering of a lotion bottle. The number of projection for the lotion bottle is 30.

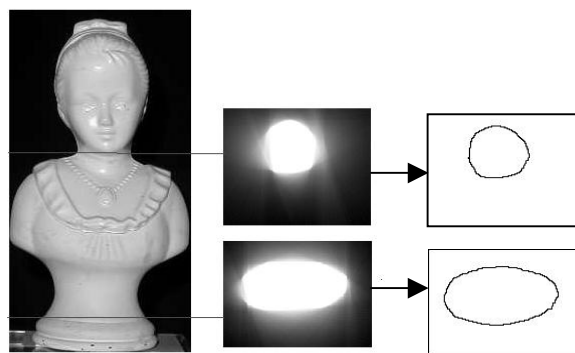


Figure 3.2 Reconstructed Cross-section Image and the Corresponding Edge-detected Image

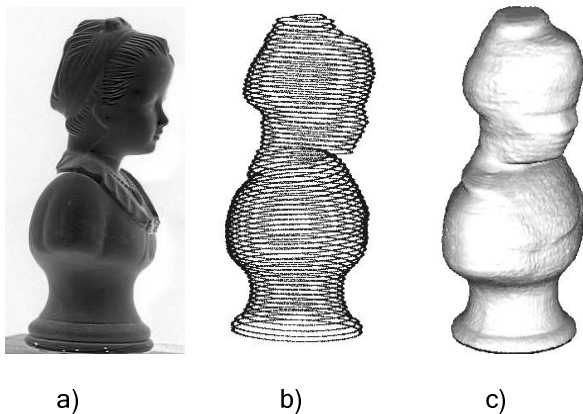


Figure 3.3 a) Original Mannequin; b) 3D Coordinate of Mannequin (Shown Every Other Slices); c) Surface-rendered Mannequin (Number of Slice: 196, Number of Projection: 60)

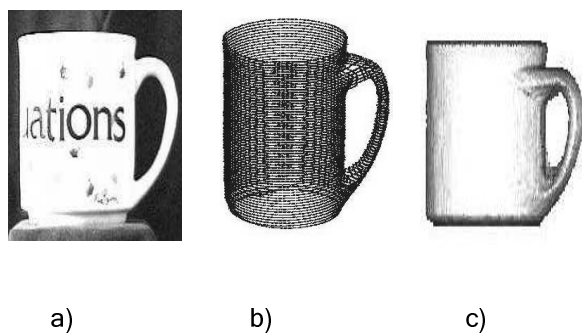


Figure 3.4 a) Original coffee cup; b) 3D coordinate of coffee cup; c) Surface-rendered coffee cup (number of slice: 62, number of projection: 72)

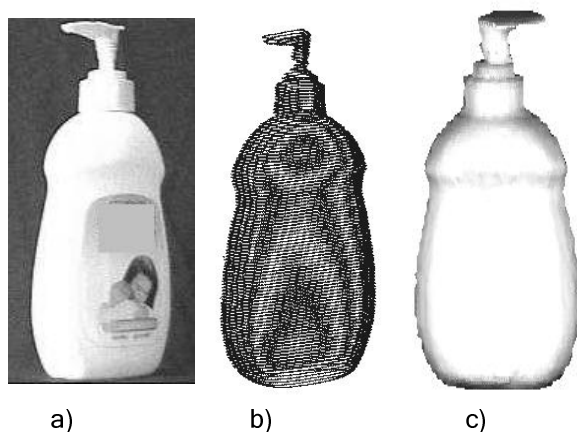


Figure 3.5 a) Origin lotion bottle; b) 3D Coordinate of Lotion Bottle; c) Surface-rendered Lotion Bottle (Number of Slice: 113, Number of Projection: 30)

## 4. Discussions and Conclusions

A new technique for 3D-shape recovery and modeling is purposed in this paper. We exploited the concept of image reconstruction from projection to reconstruct a cross-section image of the object. The cross-section image is constructed from a series of photographs taken at different angles around the object. Each of the corresponding intensity profile (row) of the digitized image resembles a projected data which can be used for reconstructing a cross-section image. To obtain the x-y coordinate of the sectional contours, we performed edge detection on the cross-section image followed by determining the location of pixel constituting the edge. Stacking of x-y coordinate yields the 3D coordinate of the object. To provide better 3D visualization, marching cube algorithm is then applied to the volumetric data, constructed from a stack of cross-section image, to extract an iso-surface. The extracted iso-surface is shading and illuminating using conventional surface-rendering technique. The advantage of our purposed technique is that it does not have to solve correspondence problem and it does not require some sort of sophisticated mechanical scanning system, which is often slow and fragile. Our technique does have two limitations. Firstly, it fails to correctly model parts of the surface of the object that are highly concave. This is because the outline of the concave part is invisible from the camera view. Secondly, the photographic process of the camera projects light on the film perspective, while the beam geometry used for the image reconstruction from projection is parallel beam. The latter drawback can be alleviated, however, by taking the photograph of the object at the farther distance (high focal length). Solution similar to the fan-beam geometry used for image reconstruction from projection needs to be addressed to solve the problem of perspective effect. Despite some drawbacks, our purposed technique is tested successfully to generate 3D-rendered surface of a variety of objects.

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