

## Anisoplanatic Image Restoration at ADFA

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**Abstract.** We have been investigating ways to restore images of objects obtained with a telescope under anisoplanatic atmospheric conditions. Anisoplanatic means that the point spread function due to atmospheric turbulence is position dependent. We began by looking at extended astronomical objects, such as craters on the moon but have turned our attention to extended objects on the earth's surface imaged in the horizontal direction in daytime. Our method involves two stages — registration to dewarp individual frames and blind deconvolution of the result. Of particular interest is that we have shown that super-resolution may result because of an apparent increase in telescope aperture due to atmospheric turbulence. A spin-off is a visualisation of the turbulence, currently used to examine the wake behind a jet aircraft. We are also investigating a multiconjugate adaptive optics solution to the problem.

### 1 Introduction

For a number of years our group has been investigating ways to restore images of objects obtained with a telescope under anisoplanatic atmospheric conditions [1–13]. In imaging through the atmosphere, variations in refractive index in turbulent air cause the light wavefront to distort, leading to a degraded image. Each point in the ideal image is moved and smeared by a point spread function or PSF. If the light over the extent of the image passes through the same patch of turbulence, irrespective of angular direction, the PSF is position invariant, and the case is considered isoplanatic. This case, which has been the most common in astronomical image processing until recently, has had considerable effort applied to its restoration. Most adaptive optics used by astronomers to date work also assume isoplanism [14–24].

#### 1.1 Isoplanatic Imaging

In the simplest case, isoplanatic distortion can be represented by the convolution

$$g = h * f + n \quad (1)$$

where  $g$  is the degraded image,  $h$  the PSF,  $f$  the true image,  $n$  is additive noise and  $*$  is the convolution operator. If  $h$  is known a priori, restoration can be achieved simply with a Wiener filter

$$\overset{(1)}{F} = \frac{H^*}{|H|^2 + \phi} G \quad (2)$$

where upper case symbols represent the Fourier transforms of corresponding lower case objects, and  $\phi$  is a parameter based on noise-to-signal ratio.

However, in astronomical isoplanatic image restoration, images are usually formed under narrow-band, quasi-monochromatic conditions, and the PSF appears as a “speckle” interference pattern. Many different methods of restoration, not based on the Wiener filter, have been devised for astronomy and the techniques are generally known as speckle restoration. These are based on phase restoration, including an iterative procedure requiring known or estimated non-zero extent of the true image (i.e., a bright object or set of objects in a surrounding black background) and phase inferred from a triple correlation or bispectrum [14–24].

## 1.2 Anisoplanatic Imaging

In the Anisoplanatic case, in which we are interested, light over the extent of an image passes through different regions of turbulence, leading to a PSF that is position dependent. In astronomical imaging, the isoplanatic patch is typically about 5 arcsec across. Thus, astronomical imaging over fields of view wider than 5 arcsec is anisoplanatic. In daytime, horizontal imaging of objects on the earth’s surface, anisoplanism may exist over even narrower fields of view, since the turbulence can be considered as “volume turbulence”, completely filling the space between the telescope and the objects being imaged.

An obvious solution to anisoplanatic restoration is to divide and conquer. A wide field-of-view image is broken into a mosaic of sub-images, such that each sub-image is approximately isoplanatic. Each is then restored assuming a PSF that is position-invariant, and the resulting sub-images are then mosaiced again into a single image. Such a technique has been used in solar astronomy, but multiple edges are a problem [25–28]. However, we have adopted a different approach, discussed in the next section.

A particularly interesting result is that we can show that super-resolution may occur because of an apparent increase in telescope aperture due to atmospheric turbulence. Such a result appears at first sight to be counter-intuitive — we are saying that the degrading turbulence may actually be beneficial! Several other researchers have pointed to this possibility [37–41]. Super-resolution is defined in this case as achieving resolution better than the diffraction-limited result, which is limited by the wavelength of the light and telescope pupil diameter  $D$ . In the anisoplanatic case, the random turbulence-induced wavefront tip-tilt may allow higher spatial frequencies than otherwise to enter the telescope pupil, making it appear larger than the actual  $D$  and thus improving the resolution by an equivalent amount. Simulations in which anisoplanatic tip-tilt distortion is applied, followed by our standard registration and dewarping motion-blur removal appear to support this idea, as does initial work using real world imagery [2].

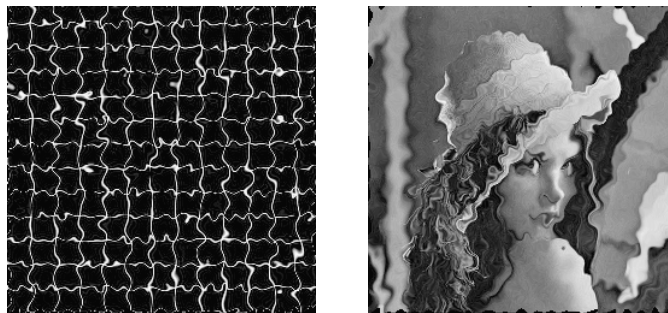
A spin-off of the first stage, registration process is a visualisation of the tip and tilt components due to the turbulent atmosphere. Thus, we are able to visualise clear air

turbulence by this method, which could be useful in choosing a site for an optical observatory, for example. We are currently investigating the use of this method of turbulence visualisation to examine the wake behind a jet aircraft.

The construction of a super fast, megapixel sensor ( $> 500$  frames/sec) based on a new CMOS photosensor chip that will enable us to “freeze” atmospheric effects during daytime, surveillance image experiments, is nearly complete [4]. In addition, we now have the ability to obtain simultaneous dual-camera image sequences using a beam splitter. This will be used in preliminary investigations into the usefulness of phase-diversity techniques in anisoplanatic restoration, as in [25–27].

On another front, we are investigating the behaviour of coated thin piezo-electric films for possible use as a deformable mirror in anisoplanatic multiconjugate adaptive optics, as in [42–43]. This work is preliminary and is part of a PhD student’s research project.

## 2. Two-Stage Approach to Anisoplanatic Restoration

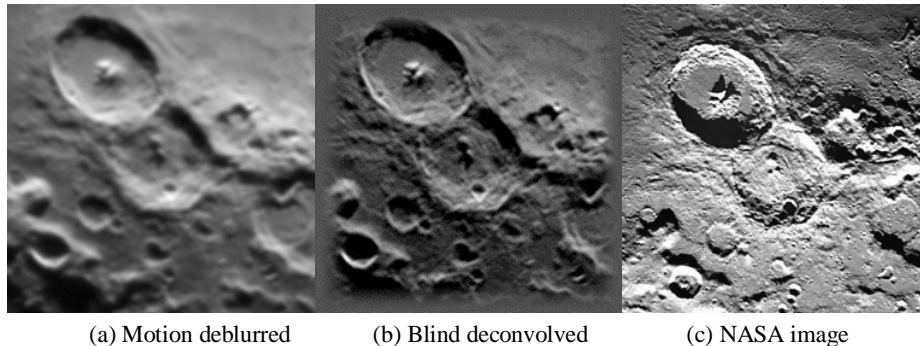


**Fig. 1.** Simulated first-order warping effect of atmospheric turbulence.

In contrast to a mosaic approach to anisoplanatic image restoration [25–28], our idea is to treat the image as a complete unit and to carry out the restoration in two, independent stages. In the first stage, we register the frames, locally warped as in Fig. 1, of a movie sequence to a motion-blurred “prototype” formed from the average of the movie sequence. The average of the frames after registration and dewarping is therefore a motion deblurred result and provides the input to the second stage. The second stage makes use of blind deconvolution, which is now possible because the remaining blur PSF has been made position-invariant.

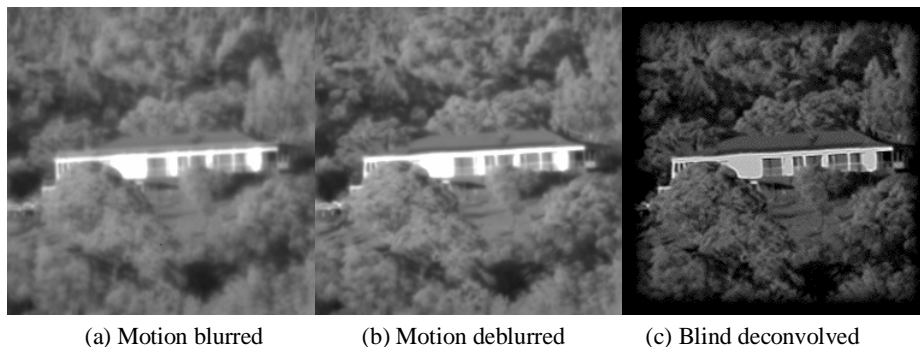
We have been experimenting with different registration methods [29–33], and blind deconvolution methods, based on several well-known techniques [34–36].

We began this work some years ago by imaging extended astronomical objects, such as craters on the moon, as in Fig. 2 [10], but have more recently turned our attention to extended objects on the earth’s surface imaged in the horizontal direction in daytime [1, 5–8], as in Fig. 3, which has application in telescope surveillance.



**Fig. 2.** Example: Lunar crater, *Theophilus*, restored by the two-stage method.

Atmospheric turbulence tends to be a lot worse under daytime, horizontal viewing conditions than in astronomy at night. If the viewing path remains close to the ground, the whole path is affected by a volume of turbulence, generated as convection currents from the solar heated ground. In extreme cases, objects in an image become totally scrambled and it is unlikely that any realistic method can restore such a result.



**Fig. 3.** Example: horizontal image, distance 10 km, restored by the two-stage method.

There is a “window of opportunity” for the two-stage approach, which exists between having perfectly still air, as in the early morning, with a clear image, and turbulent distortion enough to remove important detail from objects, allowing at least some form of the objects to be recognizable — e.g., see example in Fig. 3.

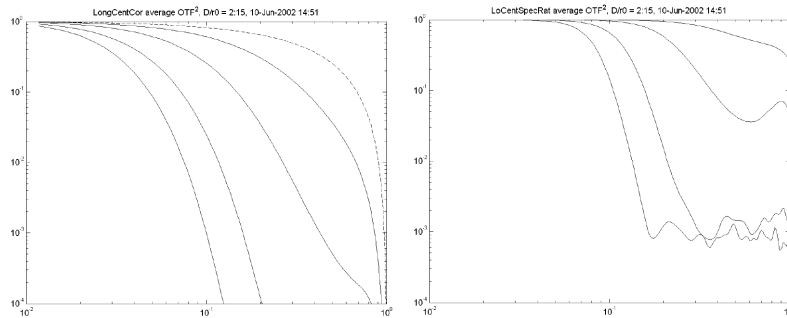
## 2.1 Comparison of Restoration with Speckle Methods

Since the result of blind deconvolution is only as good as the signal-to-noise ratio at high spatial frequencies of the motion-blur corrected stage, it is important to investigate how good this can be. For this reason, we have simulated the isoplanatic case, with a sequence of speckle point spread functions (PSFs) generated using Kolmogorov turbulence statistics [3], based on [15–16]. This allows us to compare the Optical Transfer Function (and therefore signal strength variation with spatial

frequency) for different telescope pupil diameter  $D$  and turbulence Fried parameter  $r_0$  ratios ( $D/r_0$ ) and different tip-tilt corrections [3].

It is well known that the short exposure, quasi-monochromatic OTF has a fall-off at high spatial frequencies very much less than that of the long exposure OTF. This fact is the main reason for the interest in speckle restoration during the past 30 years or so, since Labeyrie's original work [23]. Although our own area of interest, covers wide field-of-view imaging, in which the PSF varies markedly across the image, the OTF investigation is for a PSF that is position-invariant. This still fits our problem area because, after motion-blur correction and before attempting blind deconvolution, we have shown that the remnant blur PSF is effectively position-invariant [10].

To investigate the OTF, we make use of a Kolmogorov phase screen generating program [15]. The phase screen is generated for a chosen  $D/r_0$  ratio, the ratio of the telescope objective diameter,  $D$ , to the turbulence Fried parameter,  $r_0$  [24]. An example simulated result, that agrees well with that of [16], is shown in Fig. 4.



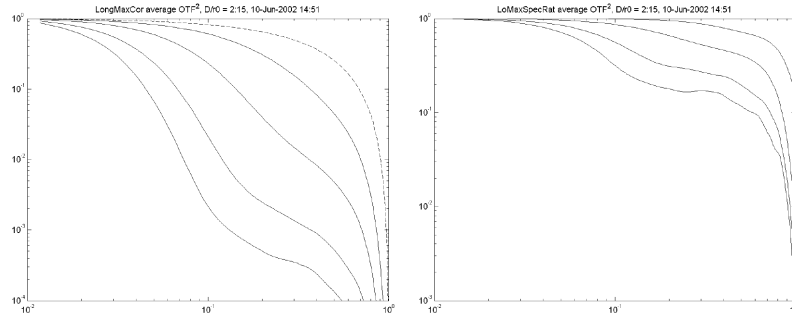
**Fig. 4.** Left: Long-exposure OTF after tip-tilt correction by centering the centroid of each speckle image, for  $D/r_0 = 2, 5, 10$  and  $15$ ; Right: Ratio of speckle, short-exposure OTF to the left-hand result, for  $D/r_0 = 2, 5, 10$  and  $15$ , plotted against spatial frequency (normalized to the diffraction-limited cutoff frequency).

It is known that a large part of the high frequency loss in long exposure imaging is due to tip/tilt effects. It can be seen that the long-exposure OTF is close to zero from quite low (normalized) spatial frequencies for  $D/r_0$  ratios corresponding to medium to large astronomical telescopes where speckle restoration is necessary. The right-hand plot of Fig. 4 is significant. It suggests that, at least for  $D/r_0 < 5$ , say, it may be possible to blind-deconvolve such a long exposure image to approach a diffraction-limited result. In our wide-area experiments, discussed in the Introduction, we do appear to be achieving good results without resorting to a speckle approach.

A very interesting result is obtained if we follow the maximum-valued pixel in the speckle image, rather than the centroid. In this case, the maximum-valued pixel is centered before averaging. The resulting long-exposure OTF2 is shown on the left of Fig. 5.

The importance of this result is that, if we can find a registration method for our two stage restoration that is equivalent to registering a maximum brightness value in a small area, then the residual blur to be removed by our second stage blind

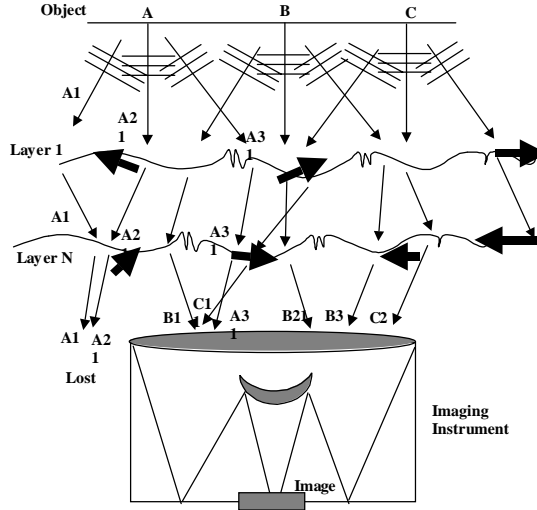
deconvolution may have a much higher signal-to-noise level than expected. If so, our two-stage method may well rival some of the more esoteric speckle approaches.



**Fig. 5.** Left: Long-exposure OTF after centering the maximum pixel of each speckle image, for  $D/r_0 = 2, 5, 10$  and  $15$ ; Right: Ratio of speckle, short-exposure OTF to the left-hand result, for  $D/r_0 = 2, 5, 10$  and  $15$ , plotted against spatial frequency (normalized to the diffraction-limited cutoff frequency).

This important question is still under investigation (i.e., how do our current registration methods behave in this case?)

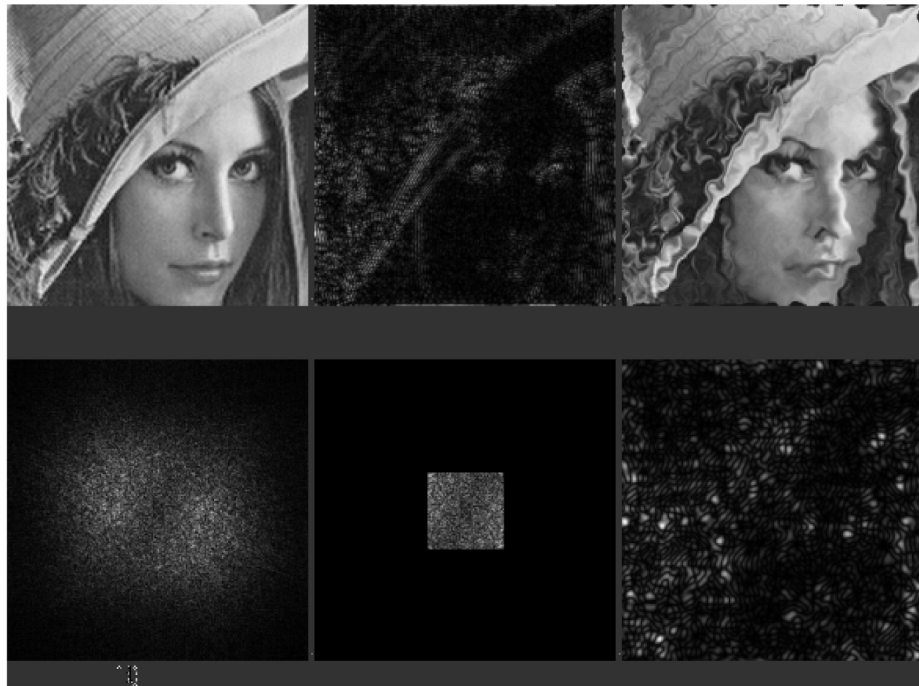
## 2.2 Super-resolution



**Fig. 6.** A simplified illustration of the anisoplanatic imaging scenario.

The variation in the refractive index of the atmosphere is modeled here as two phase screens whose structure depends on sub-regional evolution of the turbulent eddies in the atmosphere. The “scribbles” in the screens indicate a lack of correlation

between sub-regions, but for a single moment when the exposure is taken they are continuous. Note that the various rays, indicating the angular spectrum decomposition of regions of the object being imaged, will traverse slightly evolved regions of the atmosphere, so the screen is a function of from where the ray originated (and hence time of flight). The large black arrows indicate the flow directions of the eddies. Of interest is the shifting of plane wave components across the aperture, and change in their respective angles. The angular change found in the aperture is exhibited as positional shift of the image region, and the location change is an aliasing of spatial frequency content. It is this latter fact that can lead to super-resolution.



**Fig. 7.** Simulation of aliasing of super-frequency content through geometric warping. (a) 256x256 test image (lena) as low pass filtered by a simulated aperture of 64x64 pixels. (b) The image content of unwarped lena that would be lost when frequencies not passed by the aperture. (c) Geometrically warped version of lena. (d) The corresponding Fourier transform. (e) The presence of aliased frequencies is evident in the now low passed Fourier transform as would be received by the aperture, and (f) the resulting image from only aliased super-frequencies in (e).

This simulation illustrates the possibility of super-resolution due to the warping effect of tip/tilt of the wavefront due to atmospheric turbulence. Some real world examples appear to bear this out [2].

### 3. Conclusions

We have given an overview of past, present and future work of our group at ADFA. The work began with imaging extended astronomical objects, in our case, craters on the moon. We have been interested in extending this to horizontal telescopic imaging, with particular reference to surveillance. Our two stage approach, involves firstly a registration and dewarping stage, leading to a motion-blur corrected result and with a remaining position-invariant point spread function, finally restored by blind deconvolution. This approach is still unique in the image restoration community.

A spin-off of the method has been visualization of the causal atmospheric turbulence, and this is currently being put to use to investigate the wake behind jet aircraft.

We have shown that, with a suitable registration procedure, we may be able to maintain a reasonable signal-to-noise ratio through maintaining an OTF that does not drop off too rapidly with spatial frequency. Such a result is the main reason that others look to speckle restoration, with its own difficulties.

Most interestingly, we have shown that super-resolution may be possible, directly as a result of the inherently distorting turbulent effects. This agrees with philosophical arguments that, if it is possible to restore images as a result of degrading effects, the restoration may also improve the quality of the optical instrument [20, 22].

We will continue to investigate the latter two results, while also looking into an economical adaptive optics solution to the problem.

### Acknowledgements

We acknowledge the support of the University of New South Wales, the Australian Defence Force Academy, and the Australian Research Council.

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