A Hough Based Eye Direction Detection Algorithm without On-site Calibration

Takeshi Takegami*, Toshiyuki Gotoh*, Seiichiro Kagei*, and Reiko Minamikawa-Tachino**

* Graduate School of Engineering, Yokohama National University, Yokohama-city, 240-8501 Japan.
** The Tokyo Metropolitan Institute of Medical Science, Bunkyo-ku, Tokyo, 113-8613 Japan.

E-mail: *{takegami, gotoh, kagei}@sci.ynu.ac.jp, **tachino@rinshoken.or.jp

Abstract. This paper proposes an algorithm to estimate gaze direction without on-site calibration even when the position of the head relative to the camera moves. We define eye direction as geometrical direction of the eyeball and gaze direction as long fixed looking detection in this paper. In the algorithm to detect the eye direction, a cornea curvature center is detected based on a relation model between a light source and the cornea curvature center. A shape of the pupil is approximated using the detected center by ellipse Hough transform. The eye direction is estimated as a rotation angle of the approximated pupil using a cornea curvature radius and distance between the cornea curvature center and a pupil rim. This personal information on the eyeball is essential to the eye direction detection and the third personal information, a gap of the fovea, to the gaze direction detection. They are required measuring one time beforehand. However, on-site calibration is never to do in the gaze direction detection, which corrects the eye direction by the gap of the fovea. The effectiveness of our algorithm was confirmed about three point of views. The first is that the cornea curvature center was detected stably, the second that the eye direction for 4 examinees was measured with dispersion of almost same level as the involuntary eye movement, and the third that the gaze direction was detected stably moving the camera instead of the head.

Key words: Eye direction detection, gaze direction, cornea curvature center, pupil shape, Hough transform.

1. Introduction

Measurement systems for eye movement are expected to apply to various problems in human interface, medicine, psychology, and biological engineering. Especially, in medicine, eye movement have been investigated in aspects of the diagnosis of diseases such as strabismus and eye movement disability, and moreover the relation
between eye nystagmus and diseases such as vestibular neuritis and Minkhreis disease.

Gaze measurement begun to be researched using electricity in the late 18th century. Nowadays, gaze is detected using eye images captured though a camera. There are a various styles of detection such as holding the head in a place, head-free wearing a glass with a target or wearing a head mounted camera, and with/without calibration. However, the gaze direction had been estimated using the eyeball center in almost all styles. We also proposed a self-calibration algorithm holding the head in a place, that did calibration using images obtained by useless special targets [1]. Furthermore, we proposed an improved algorithm to detect a shape of the pupil by ellipse Hough transform using parameters obtained by the self-calibration algorithm [2]. In the gaze direction detection using the eyeball center, the suggested causes of errors were that the eyeball center could not be estimated directly and that rotation of the eyeball was not always simple motion.

Therefore, the cornea curvature center instead of the eyeball center has been used in the gaze direction detection [3]-[5]. The advantage of using the cornea curvature center provides us with head-free detection. Because it can be estimated directly using images reflected on the cornea curvature. As the latest research of related works, Ohno et al. proposed an algorithm to detect gaze as direction of a vector from the cornea curvature center to the pupil center and the cornea curvature center was detected using the focus distance of a camera [6]. There still exist some problems. That is errors might be generated by short length between the cornea curvature center and the pupil center (cf. Fig. 1). The length is about 5 mm and errors of 0.1 mm in center detection for the cornea curvature or the pupil generate approximately gaze direction error of 1 deg. And then it has been not confirmed that the cornea curvature center is an unique center point for rotation of the cornea curvature.

Therefore, for gaze direction detection with high precision, it is required to exactly detect eye direction distinguished from gaze direction. The eye direction is defined as geometrical direction of the eyeball and the gaze direction as long fixed looking detection. We propose an eye direction detection algorithm using a rotation angle of a flattened ellipse of the pupil. In this algorithm, the cornea curvature center is used indirectly and the pupil is approximated robustly using Hough transform. Although personal information is required measuring one time beforehand, on-site calibration is required no any more. Moreover, it allows us to detect eye direction even when the position of the head relative to the camera moves.

**Figure 1.** An eye direction model based on a flattened shape of the pupil in an obtained image.
2. An eye direction model based on a flattened shape of the pupil

2.1 Overview

The eye direction is detected using an image captured an eye in our model that includes an eyeball model as shown in Fig. 1. Eye direction is calculated using the pupil center and the cornea curvature center instead of the eyeball center in the eye direction model. A shape of the projected pupil is approximated as an ellipse flattened by a rotation angle of the pupil. The flattening is provided with the eye direction as a rotation angle of the pupil. However, examineeis personal information of the eyeball is essential to the eye direction detection, which consist of a cornea curvature radius, a distance between the cornea curvature center and the pupil rim, and a gap of the fovea. Once they are measured as absolute values, on-site calibration is not required no any more for our eye direction model.

The eye direction detection constructs three steps. First, a tentative cornea curvature center is approximated using cornea reflections from four LEDs around a camera lens and the cornea curvature radius measured as the personal information. Next, a shape of the pupil is approximated as an ellipse by Hough transform using outline edges of a projected pupil in the image. Ordinarily, an ellipse Hough transform is performed using 5 degree of freedom as parameters. Our eye direction model enables us to reduce the degree of freedom to 3. That is only the pupil center and the pupil radius. Approximation of the pupil shape by ellipse Hough transform is also performed varying the cornea curvature center. And then, the eye direction is estimated by calculating the rotation angle of the pupil.

2.2 Approximation of the cornea curvature center

Relation model between a light source and the cornea curvature center is shown in Fig. 2. On the assumption that \( r \) is the already-measured cornea curvature radius, a ray from a light source \( L_c \) reflects an eye image \( R_i \) through the camera \( C \). Let \( P_{ri} \) be the cornea reflection at that time. The normal vector at \( P_{ri} \) is given as the cornea normal vector \( N_{ri} \). Therefore, a cornea curvature center can be defined as \( O_{ri} \). Although positions of the cornea reflection have not fixed in actual, a locus of cornea curvature center is satisfied with the equation (1).

![Figure 2. A relation model between a light source and a cornea curvature center.](image-url)
\[ O_i(t) = P_i(t) - r \cdot N_i(t), \] (1)

where \( t \) is a parameter to regulate positions of the cornea reflection.

Using this, if two or more light sources exist, the cornea curvature center could be theoretically determined as an intersection point of each locus of the cornea curvature center. However, the intersection cannot necessarily be obtained by errors of calculation or digitization. In the case of two light sources, the distances between loci of the cornea curvature centers obtained from each light source can be calculated. A cornea curvature center is determined by the value of \( t_i \) when the equation (2) is the minimum.

\[ S = \sum_{i} \sum_{j} \| P_i(t_i) - O_j(t_j) \|^2 \] (2)

The calculated cornea curvature center is used as an initial value in Hough transform in order to detect a shape of the pupil.

2.3 Approximation of the pupil center in the eye direction model

In eye direction detection, it is required to approximate a shape of the pupil and above all a pupil center is essential. In the eye direction model, the pupil center is approximated using Hough transform. It assumes that edge points of the reflected pupil are detected in an obtained image. It is considered a single isolated edge point. There could be an infinite number of ellipses that could pass through this point. If these ellipses are generated with the identical shape, centers of these ellipses could also generate another ellipse. There could be ellipses with two layers. Figure 3 shows that the second layer ellipse is generated by centers of the first layer ellipses. In Fig. 3 (a) where an edge point is on the cornea curvature center, the second layer ellipse is a circle. This circle is obtained as following,
\[
\begin{align*}
    x &= b_x \cdot \cos \alpha \\
    y &= b_y \cdot \sin \alpha \\
    z &= r_y \\
    b_x &= a_x \sqrt{r_0^2 - a_x^2} / r_0
\end{align*}
\] (3)

where \(b_x\), \(r_y\), \(a_x\), and \(r_0\) are this circle radius, a distance between the cornea curvature center and the second layer ellipse, a pupil radius, and the distance between the cornea curvature center and the pupil rim, respectively.

In Fig. 3 (b), an edge is at an arbitrary position. It is considered that the edge point in Fig. 3 (a) is rotated about the \(\gamma\)-axis by an angle \(\beta\) and about the \(\zeta\)-axis by an angle \(\gamma\). This second layer ellipse is obtained as following,

\[
\begin{align*}
    x &= b_x \cdot \cos \alpha \sqrt{\frac{r_0^2 - x^2 - y^2}{r_0}} - b_y \cdot \sin \alpha \sqrt{\frac{r_0^2 - x^2 - y^2}{r_0}} + r_0 \frac{x}{r_0} \\
    y &= b_y \cdot \cos \alpha \sqrt{\frac{r_0^2 - x^2 - y^2}{r_0} + b_y \cdot \sin \alpha \sqrt{\frac{r_0^2 - x^2 - y^2}{r_0} + r_0 \frac{y}{r_0}} \\
    z &= -b_x \cdot \cos \alpha \sqrt{\frac{r_0^2 - x^2 - y^2}{r_0}} + r_0 \frac{z}{r_0} \\
    b_x &= a_x \sqrt{r_0^2 - a_x^2} / r_0
\end{align*}
\] (4)

where \((x, y)\) is a given edge point and others the same in equation (3).

Therefore, approximation of the pupil center by the first layer ellipses resolves itself to approximation by the second layer ellipse using Hough transform. The pupil radius is essential to equation (4) in the second layer ellipse. Moreover, the cornea curvature center is varying toward depth from the camera for 3-dimensional approximation with high precision. The Hough transform by the second layer ellipse is performed only varying the pupil radius and the cornea curvature center.

Furthermore, the second layer ellipse is used as a vote by drawing it using perspective projection.

2.4 Eye direction based on a rotation angle of an approximated ellipse of the pupil

The relation between the pupil rotation angle and the eye direction is shown in Fig. 4. The pupil is detected to rotate about \(\gamma\)-axis by an angle \(\beta\) and about \(\zeta\)-axis by an angle \(\gamma\) at the cornea curvature center. The eye direction is defined at the camera coordinate to rotate the pupil center

![Figure 4. The relation between the rotation angle and the eye direction.](image-url)
about horizontal direction by an angle \( \theta \) and about vertical direction by an angle \( \varphi \).

By this relation, angles of \( \theta \) and \( \varphi \) are defined using \( \beta \) and \( \gamma \) as following.

\[
\begin{align*}
\tan \theta &= \tan \beta \cos \gamma \\
\tan \varphi &= \tan \beta \sin \gamma
\end{align*}
\]

(5)

3. Algorithm

The algorithm configuration is shown in Fig. 5. At first, four cornea reflections of LEDs are detected in an input image. The reflections with high intensity are around the pupil and the iris whose intensity is low. Edges are detected in only high brightness regions of the image processed by smoothing. Then, the maximum intensity of the edges is detected, and edge points with 70% and over of the maximum value are defined as a cornea reflection region. If the number of cornea reflection region is not four, this process is stopped and another process is started with a next image. Otherwise, a cornea reflection is calculated as a gravity of the region, and then a tentative cornea curvature center is calculated based on Section 2.2. A shape of the pupil is approximated using ellipse Hough transform by second layer ellipses described in Section 2.3. The procedure of Hough transform is as follows:

**Step 1:** For efficient processing, outline edges of a pupil are detected by a filter size of 3 \( \times \) 3 in a square region with the size of \( 2a_0 \times 2a_0 \) (\( a_0 \): a pupil radius) centered at the tentative cornea curvature center \((x_0, y_0)\). The pupil radius \( a_0 \) has already been estimated roughly on the image. The edges of cornea reflection regions are deleted.

**Step 2:** A tentative center \((x_p, y_p)\) of the pupil is decided with the obtained edge points.

**Step 3:** An array for votes is initialized.

**Step 4:** Each edge point votes to the array by drawing a second layer ellipse to pass through itself by perspective projection. Also at this time, for efficient
processing, the ellipse is drawing only for the region of ±20 deg. centered at a direction which connects the tentative center \((x_p, y_p)\) and the edge point.

**Step 5:** After voting is finished for all edge points, the maximum of votes is detected to scan the array.

**Step 6:** Increasing \(a_0\) by 0.1 mm from -0.5 mm to 0.5 mm and a depth direction value of the tentative cornea curvature center by 0.5 mm from -2 mm to 2 mm, the operations of **Step 3**, **Step 4**, and **Step 5** are repeated. An optimized pupil radius \(a_{0\text{max}}\) and an optimized cornea curvature center \((x_0, y_0, z_0)_{\text{max}}\) are detected when the maximum of all votes is obtained.

**Step 7:** Scanning votes with \(a_{0\text{max}}\) and \((x_0, y_0, z_0)_{\text{max}}\) in the array, a high vote region is detected and the pupil center is calculated as the gravity of the region. Then, \(a_{0\text{max}}\) is substituted for \(a_0\) and that the pupil radius is updated.

And then, the eye direction is estimated using the rotation angle of the approximated pupil based on the eye direction model.

### 4. Feasibility study and discussions

**Figure 6.** The evaluation system.

#### 4.1 Configuration of an evaluation system

An evaluation system used for feasibility study is shown in Fig. 6. Figure 6 (a) shows camera setting with four LEDs around the lens. The LED is EL-1K3 (Kodenshi corp., Kyoto, Japan) with forward current 100mA, reverse voltage 5V, peak emission wave length 940 nm, and half angle ±36 deg. The evaluation was performed on DELL Dimension 8200 (CPU: Pentium IV, Clock: 1.80 GHz) with a frame grabber imagination PXC-200AL (CyberOptics, OR, USA). The algorithm was implemented in C onLinux.

Figure 6 (b) shows output display in eye direction detecting. The evaluation system is designed to instantaneously confirm intermediate results of image processing and the eye direction angles.
4.2 Image processing for pupil detection

Image processing for pupil detection is shown in Fig. 7 where eye regions are enlarged. The size of an input image is 640 x 480 pixels with 8 bits gray levels. By the lighting of infrared LEDs, a pupil is observed to be darker and larger than its surroundings and four bright regions of the LEDs can be seen in the pupil. Figure 7 (b) shows only four bright regions detected by image processing. Those bright points are used to calculate a cornea curvature center. Figure 7 (c) shows outline edges of the pupil, votes by Hough transform, a black cross as a point with the maximum of votes, and a white cross as the optimized cornea curvature center. Figure 7 (d) shows the detected pupil center as a small cross and the optimized cornea curvature center as a big cross.

4.3 Detection of a cornea curvature center

Evaluations were performed to detect a cornea curvature center based on the relation model between it and a light source described in Section 2.2. An examinee was an adult with normal eyesight and sat down holding his head in a place at the front of the screen. There appeared to be nine targets combing positions bi-directionally at -10 deg., 0 deg., and 10 deg. His cornea curvature centers were detected every 10 times when he looked at each target. An example of the detection result is shown in Fig. 8. It was confirmed that the cornea curvature center was almost measured stably for each direction. The coordinates on Z-direction are measured more unstably than on X- and Y-directions. By the way, the cornea curvature center is suggested to move according to the eye direction in the eye direction model. It was confirmed that movement of the cornea curvature center was detected synchronizing with eye direction. And also, it is confirmed that optimization of the cornea curvature center is

![Figure 7. Image processing for pupil detection.](Image 279x188 to 373x259)

![Figure 8. An example result of the cornea curvature center detection.](Image 279x283 to 472x354)
significant for the eye direction detection.

![Diagram](image_url)

**Figure 9.** An example result of the eye direction detection.

camera moved with 10 mm left, 10 mm back, and 5 mm up. As the examinees had been measured the personal information of their eye beforehand, no more calibration procedure was performed every times of moving camera. Eye direction was also measured with dispersion of almost same level as the involuntary eye movement at each target for all camera position. The average of the resultant gaze direction that was corrected eye direction by the gap of the fovea is plotted for each target in Fig. 10. For four camera position, the eye direction are nearly equal, but are dispersed at the lower targets about vertical direction and the targets of 10 deg. at horizontal direction. This dispersion might be caused by character of the examinees. It is confirmed that the eye direction was detected not depending on camera position without on-site calibration.

![Diagram](image_url)

**Figure. 10.** An example result of the gaze direction detection moving the camera instead of the head.
5. Conclusion

We have discussed the stable eye direction detection algorithm that does not need the on-site calibration even when the position of the head relative to the camera moves. At first, a cornea curvature center is detected by four cornea reflections of LEDs on the relation model between a light source and a cornea curvature center. Using ellipse Hough transform, a pupil shape is approximated precisely by the calculated center to be considered tentative. The eye direction is estimated as the rotation angle of the pupil using the approximated pupil center and the optimized cornea curvature center in the eye direction model. As the result of the feasibility study with 4 examinees, the effectiveness of our algorithm was confirmed that the eye direction was measured with dispersion of almost same level as the involuntary eye movement although their heads were fixed and the camera moved 10mm along x-, z-, and 5mm along y-direction. The algorithm allows us to detect the head-free gaze direction.

The eye direction was detected seven images per second in the feasibility study. It is suggested that our algorithm is adequate to real time processing in application with gaze direction detection. The accuracy would be confirmed by increased the examinee. Moreover, the future work is to develop systems for medical fields and a man machine interface by extending this algorithm.

References