Testing for Curves in a Binary Image

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Abstract. Curve detection is viewed as a process of hypothesis generation and hypothesis testing. Of the two, hypothesis generation has received much attention and many sophisticated post-processing strategies are published in the literature. In this work, the emphasis is shifted to the development of an efficient and effective hypothesis testing strategy to relieve the hypothesis generation from sophisticated computations. The proposed method recasts edge pixels in a binary image into a one-parameter system derived from the hypothesis. The recasting process creates a histogram, which contains a single dominant peak if and only if the hypothesis under testing contains a significant number of edge pixels. Experiments with circle testing show that the proposed strategy outperforms the global threshold.

1 Introduction

Curve detection is an elementary step in extracting shape information from a binary image. In principle, curve detection can be viewed as a process of *hypothesis generation*, which finds instances of curves that are likely to be in the image, and *hypothesis testing*, which verifies that these instances do indeed exist. While the strategy for hypothesis generation varies for different types of curve detectors, (e.g., evidence accumulation in Hough transforms [1] and consensus set computation in RANSAC methods [2, 3],) hypothesis testing is the common step within a curve detection method to finally decide which curves to accept.

In Hough transforms, hypothesis generation and testing are closely coupled in that the latter is conducted over the accumulators filled by the former. Ideally, the accumulators should have the property that peaks contributed by true positives are easily distinguishable from those due to false positives. In the ideal case, it is straightforward to devise a high confidence threshold, which makes use of simple statistics or prior knowledge, that accepts true positives and rejects false positives. In practice, the fact that Hough transform is a one-to-many mapping from a discretized image space to a discretized parameter space inevitably implies the certainty for peaks to split and merge in the parameter space. Thus, some form of a peak-sharpening post-processing strategy is usually involved in high performance Hough transforms, see the two surveys on Hough transform [4, 6]. Post-processing strategies increase computational complexity and difficulty in implementation. For example, the peak sharpening strategy of Gerig and Klein [5] is accomplished by doing the Hough transform twice: the first pass does the standard accumulation process and the second pass assigns each edge pixel to the highest-counting cell that is intersected by its hyper-surface in the parameter space. The result is strongly sharpened peaks that allows true positives to be distinguished from false positives, but at the cost of doubled computational effort of a standard Hough transform.

Alternatively, hypothesis testing can be treated independently from hypothesis generation. As two loosely coupled components, hypothesis testing requires hypothesis generation to supply no more than the parameter vectors of candidate curves. A consequence of loose coupling is that an independent hypothesis testing strategy can be used as a post-processing strategy to less sophisticated but efficient curve detectors to improve performance. An example of independent hypothesis testing is the global threshold, which counts the number of edge pixels in the input image on the loci of a curve instance and compares it against a pre-determined threshold. The global threshold is simple to implement, low in computational complexity, and can be highly effective when supplied with prior information on threshold setting. The principal disadvantage is in its lack of statistical support in making an acceptance decision.

In this paper, an independent hypothesis testing strategy for curve verification is proposed. The objectives are to provide a sound statistical foundation for making acceptance and rejection decisions while keeping it comparable to the global threshold in computational complexity and implementation simplicity. A brief overview of the proposed strategy is as follows. To test an instance of curve under detection, a one-parameter system of curves, which the instance is a member of, is constructed. A transformation based on the one-parameter system is created and applied to edge pixels in the image; the result is a one-dimensional distribution-counting histogram of the edge pixels in the image against a range of specific members of the one-parameter system. The instance is accepted if and only if it coincides with the single dominant peak in the histogram.

2 Hypothesis testing

Let h be an instance of a curve (e.g., a line, a circle or an ellipse) found by a hypothesis generation strategy such as Hough transform or RANSAC. The instance h can be represented as a member of a one-parameter system of curves:

$$s(\lambda) = (1-\lambda)s_1 + \lambda s_2 = 0, \lambda \in \Re, \tag{1}$$

where $s_1 = 0$ and $s_2 = 0$ are two distinct curves of the same type as h and intersect h at one, two, or four points, respectively, if h is a line, a circle, or an ellipse, respectively.

Edge pixels (x, y) in the image that h is extracted from can be mapped into a one-dimensional parameter space with the function $\lambda_{s_1 s_2} : \Re \times \Re \to \Re$ derived from Equation (1):

$$\lambda_{s_1 s_2}(x, y) = \frac{s_1(x, y)}{s_1(x, y) - s_2(x, y)}. \tag{2}$$

Computationally, the mapping creates a histogram of the numbers of edge pixels in the image coinciding with the loci of the curves of the system against a range of λ. Thus, if h is indeed a true positive, the histogram is expected to have a well-defined peak since edge pixels on h are mapped into the same value by the function $\lambda_{s_1 s_2}$. Furthermore, we claim that no curves other than h or noises are likely to produce a well-defined peak in the histogram. Let j be a curve that is not a member of the one-parameter system constructed from h. Edge pixels on j are distributed into the histogram such that each member in the one-parameter system receives at most $\deg(j) \cdot \deg(h)$ votes from j, where deg is the algebraic degree of a curve. For outliers categorized as uniformly distributed noise with noise level γ, the number of noise edge pixels accumulated in a bin is a binomial random variable [3] with an expected value of $\gamma B(\lambda)$, where $B(\lambda)$ is the number of pixels on the curve $s(\lambda)$.

The above analysis allows one to conclude that in the histogram over a properly chosen range of λ, the presence of the single dominant peak indicates that the hypothesis is statistically more likely to be a true positive than other curves. This property allows us to devise an adaptive threshold; an example will be given in Section 2.1. In contrast, when a hypothesis is accepted by a global threshold, there is no statistical support for claiming that it is more likely to be a true positive than other curves.

The proposed hypothesis testing strategy involves the following steps. (1) Determine the range of λ and the base curves $s_1 = 0$ and $s_2 = 0$ from the instance h and the input image. (2) Accumulate the histogram by applying the function $\lambda_{s_1 s_2}(x, y)$ to edge pixels in the image. (3) Detect the existence of a single dominant peak in the histogram. The time complexity of the proposed strategy is as follows. The construction of base curves and the transform takes constant time. The accumulation for testing a hypothesis takes $O(n)$ time, where n is the number of edge pixels in the input image. The statistics computation takes $O(m)$ time, where m is the number of bins in the histogram. Since $m < n$, the overall computational complexity is $O(n)$, which is asymptotically the same as that of the global threshold.

Next, the process is illustrated with circle testing.

2.1 Circle testing

Construction of base curves. Let h be a circle with parameter (x_0, y_0, r_0) found by a hypothesis generation strategy from an image I. First, to limit the range of interest of λ, we want to choose the base curves s_1 and s_2 so that $\lambda(s_1) < \lambda(h) < \lambda(s_2)$. One way to achieve this is to let the base curves have a radius larger than that of the hypothesis. For example, by choosing base radius to be $\sqrt{2} r_0$ and designating $\lambda(s_1) = 0$, $\lambda(s_2) = 1$ and $\lambda(h) = \frac{1}{4}$, the one-parameter system (coaxal system [8]) is defined

$$(1-\lambda)((x-x_0+\frac{1}{\sqrt{3}}r_0)^2+(y-y_0)^2-2r_0^2)+\lambda((x-x_0-\sqrt{3}r_0)^2+(y-y_0)^2-2r_0^2)=0. \quad (3)$$

The construction is illustrated in Figure 1.

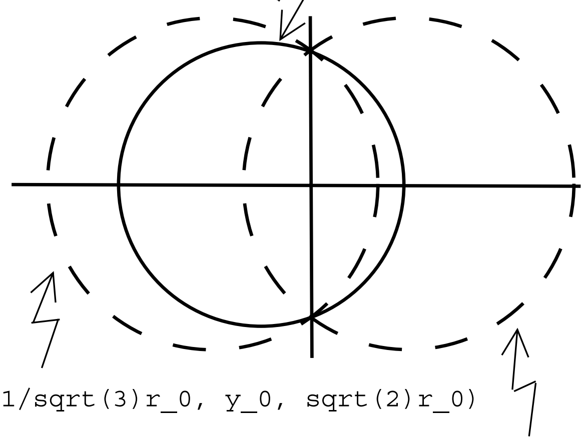

Fig. 1. Coaxal system of circles

Computing histogram. First, the number of bins in the histogram, which covers the range of λ values in $[0,1]$, is decided. One way to do this is to set the number of bins in proportion to the distance between the centers of the base curves. The transform defined from coaxal system in (3) is

$$\lambda_{s1s2}(x,y)=\frac{(x-x_0+\frac{1}{\sqrt{3}}r_0)^2+(y-y_0)^2-2r_0^2}{\frac{8}{3}(\sqrt{3}(x-x_0)r_0-r_0^2)}. \quad (4)$$

Edge pixels in the image is mapped into the histogram. Note that λ values for edge pixels mapped outside $[0,1]$ are discarded.

Detecting the single dominant peak in histogram. Based on analysis for the existence of a single dominant peak in case of a true positive, an adaptive threshold for hypothesis testing can be devised. First, the sample mean $\overline{X}$ and sample variance S^2 [9] of the histogram except the peak are calculated. The hypothesis is accepted if and only if its λ value coincides with that of the highest peak which has a count greater than $\overline{X}+wS$, where $w > 0$ controls the final threshold setting. A large w ensures a low false positive rate at the cost of a higher missing rate. A small w ensures a low missing rate at the cost of a high false positive rate. For a given w, the actual threshold adapts to the complexity of image since the sample mean and the sample variance change as image complexity changes. In contrast, the fixed threshold used in the global threshold is more sensitive to the image complexity.

3 Experiments

In this section, the performance of the proposed hypothesis testing strategy is compared with the global threshold in the context of circle detection. The standard Hough transform for circle is used for hypothesis generation. The test images are synthetic images containing three concentric circles of varying degrees of completeness in the presence of uniform pepper-and-salt noise. Table 1 summarized the parameters used to generate the test images.

Table 1. Parameters used to generate the test images

image size	50×50
noise levels	0%, 1%, 2%, 3%, 4%, 5%, 6%, and 7%
circle 1	center: (25,25), radius: 10; 30% of edge pixels missing
circle 2	center: (25,25), radius: 15; 40% of edge pixels missing
circle 3	center: (25,25), radius: 20; 50% of edge pixels missing

To compare the performance of the proposed method with the global threshold, the following definitions are used.

- N_h: number of hypotheses generated by the standard Hough transform.
- w: the threshold control of the proposed method.
- $\bar{n}$: the threshold control of the global threshold, defined as the number of edge pixels on the loci of a circle in the image normalized by its circumference.
- Φ : range of operation. The range of thresholds that a hypothesis testing strategy generates non-null results. For example, $[0, 0.6]$ is the range of operation for the global threshold if all hypotheses are rejected by a threshold greater than 0.6.
- $\Phi_{\texttt{ideal}}$: ideal range of operation. The sub-interval of the range of operation such that a threshold set in this sub-interval accepts all true positives and rejects all false positives.
- $\phi_{\texttt{ideal}}$: coverage of ideal range of operation. The width of the ideal range of operation divided by the width of the range of operation, i.e., $\phi_{\texttt{ideal}} = \frac{|\Phi_{\texttt{ideal}}|}{|\Phi|}$. In general, the larger $\phi_{\texttt{ideal}}$ is, the easier it is to separate true positives from false positives.

- F: number of false positives accepted by the maximum threshold which accepts all true positives.
- $t_{\texttt{sht}}$, $t_{\texttt{proposed}}$, and $t_{\texttt{global}}$: the execution time of the standard Hough transform, the proposed hypothesis testing strategy, and the global threshold, respectively.

The results of executing the proposed hypothesis testing strategy and the global threshold against the test images are shown in Table 2 and Table 3. Note that in proposed method, the hypothesis to test is located at $\lambda = 0.25$ and the sample means and the sample variances are computed over $[\lambda = 0, \lambda = 0.5]$. As can be seen, the proposed strategy performs consistently better than the global threshold at all noise levels.

Table 2. Test results for the proposed strategy

image at noise level	N_h	Φ	$\Phi_{\texttt{ideal}}$	$\phi_{\texttt{ideal}}$	F
0%	3	$[0, 10.53]$	$[0, 8.56]$	0.81	0
1%	4	$[0, 9.91]$	$[4.20, 7.80]$	0.36	0
2%	5	$[0, 10.23]$	$[3.11, 7.71]$	0.45	0
3%	13	$[0, 8.67]$	$[4.32, 7.04]$	0.43	0
4%	20	$[0, 8.24]$	$[4.00, 6.41]$	0.29	0
5%	30	$[0, 7.95]$	$[3.51, 5.32]$	0.23	0
6%	40	$[0, 8.21]$	$[4.11, 4.67]$	0.07	0
7%	62	$[0, 7.36]$	-	-	1

Table 3. Test results for the global threshold

image at noise level	N_h	Φ	$\Phi_{\texttt{ideal}}$	$\phi_{\texttt{ideal}}$	F
0%	3	$[0, 0.67]$	$[0, 0.46]$	0.69	0
1%	4	$[0, 0.69]$	$[0.36, 0.46]$	0.14	0
2%	5	$[0, 0.69]$	$[0.34, 0.46]$	0.17	0
3%	13	$[0, 0.73]$	-	-	1
4%	20	$[0, 0.73]$	-	-	2
5%	30	$[0, 0.77]$	-	-	4
6%	40	$[0, 0.78]$	-	-	5
7%	62	$[0, 0.8]$	-	-	13

The comparison of execution times is in Table 4. In all test images, the proposed method uses slightly more time than the global threshold. Note that both hypothesis testing strategies consume only a negligible amount of time in comparison with the standard Hough transform.

Table 4. Execution time in seconds

image at noise level	$t_{\texttt{sht}}$	$t_{\texttt{proposed}}$	$t_{\texttt{global}}$
0%	0.651191	0.00124	0.001023
1%	0.778455	0.001684	0.001419
2%	0.864781	0.002129	0.001785
3%	0.912859	0.005546	0.004701
4%	0.998567	0.00883	0.007422
5%	1.12201	0.01335	0.01138
6%	1.14211	0.05721	0.05486
7%	1.27379	0.06555	0.06524

4 Concluding remarks

A hypothesis testing strategy for use in curve detection is proposed. The strategy recasts edge pixels in an image into a one-parameter system. Computationally, the proposed strategy is as efficient as the global threshold. Experiments on circle testing show that it not only outperforms the global threshold strategy, but also exhibits a wider and relatively stable ideal range of operation. Requiring no more a collection of hypotheses as the input, it can be used as a post-processing strategy to the standard Hough transforms [1], randomized Hough transforms [7], and RANSAC based methods [2, 3, 10]. The implementation of testers for lines and ellipses is not difficult and is underway.

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