Automorphism Groups of Certain Non-Quasiprimitive Almost Simple Graphs

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Abstract

A graph Γ is G-quasiprimitive if G is a group of automorphisms of Γ such that each nontrivial normal subgroup of G acts transitively on the vertices of Γ. We consider Γ a finite, connected S-locally-primitive graph with S a nonabelian simple group and give a set of conditions under which we may guarantee that either the full automorphism group of Γ is not quasiprimitive or there is a non-quasiprimitive subgroup Y of $\mathrm{Aut}\Gamma$ such that $C.G$ is maximal in Y, where C is the centralizer of S in $\mathrm{Aut}\Gamma$ and G is an almost simple group with socle S. As an application of this result we show that, in certain circumstances, $\mathrm{Aut}\Gamma = Z_p.G$, where p is a prime and $G = \mathrm{Aut}\Gamma \cap \mathrm{Aut}(S)$.

1 Introduction

A permutation group on a set Ω is said to be *quasiprimitive* if each of its nontrivial normal subgroups is transitive on Ω. By the structure theorem in [18, Section 2] quasiprimitive permutation groups may be divided into 8 disjoint types (HA, HS, HC, AS, SD, CD, TW, PA) in a way which is helpful for applications to graph theory (see the description in [3]). The main type considered in this paper is AS: a quasiprimitive group G is said to be of type AS if $S \leq G \leq \mathrm{Aut}(S)$, for some nonabelian simple group S.

Let Γ be a finite simple undirected graph with vertex set $V\Gamma$ and edge set $E\Gamma$ and let G be a group of automorphisms of Γ. The graph Γ is said to be G*-quasiprimitive* if and only if G is quasiprimitive on $V\Gamma$. In particular, an $\mathrm{Aut}\Gamma$-quasiprimitive graph is said to be *quasiprimitive*. The graph Γ is said to be G-*locally-primitive* if G is transitive on $V\Gamma$ and the stabilizer G_α is primitive on

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$\Gamma(\alpha)$, where $\Gamma(\alpha)$ denotes the set of all vertices adjacent to α. The graph Γ is said to be $(G,2)$*-arc transitive* if G is transitive on the 2-arcs of Γ.

Praeger in [19] raises the questions: (1) for a G-quasiprimitive graph Γ, under what conditions can we be certain that $\mathrm{Aut}\Gamma$ is quasiprimitive on $V\Gamma$? (2) If $\mathrm{Aut}\Gamma$ and G are quasiprimitive with the same quasiprimitive type, is it possible that $soc(G) \neq soc(\mathrm{Aut}\Gamma)$, and if so what are the possibilities for these socles?

We look first at question (1). It is shown in [7] that, for a connected G-quasiprimitive graph Γ with G of type AS, if Γ is G-locally-primitive then either Γ is quasiprimitive or $\mathrm{Aut}\Gamma$ has a restricted structure. Baddeley in [2, Section 6] constructs the first example of connected non-quasiprimitive graph Γ which is $(G,2)$-arc transitive, for some quasiprimitive group G of type TW, and he comments there that such graphs seem difficult to construct. An infinite family of $(\mathrm{L}_2(q),2)$-arc transitive graphs with valency 4 is constructed in [12], and Li [14] proves that the full automorphism groups are isomorphic to $Z_2\times \mathrm{L}_2(q)$, which gives the first infinite family of non-quasiprimitive graphs Γ such that $\mathrm{L}_2(q)$ acts quasiprimitively on $V\Gamma$. Recently an infinite family of $\mathrm{U}_3(q)$-quasiprimitive transitive graphs of valency 9 has been constructed by the authors of this paper in [8]; their full automorphism groups are isomorphic to $Z_3.G$ with $Z_3 = \mathrm{C}_{\mathrm{Aut}\Gamma}(\mathrm{U}_3(q))$ and $\mathrm{U}_3(q) \leq G \leq \mathrm{Aut}(\mathrm{U}_3(q))$. From this we obtain a new infinite family of non-quasiprimitive graphs Γ admitting a quasiprimitive group of type AS acting transitively on the vertices and the 2-arcs of Γ. Observing all non-quasiprimitive examples with G quasiprimitive given above, we find that $\mathrm{Aut}\Gamma = C.G$ with C some small nontrivial cyclic group, which motivates us to suggest a set of conditions under which we may guarantee that either Γ is non-quasiprimitive or $\mathrm{Aut}\Gamma$ has a restricted structure. In this paper we concentrate on G-quasiprimitive graphs for G of type AS. The set of conditions we employ is the following

Condition $\mathcal{P}$. *Let Γ be a connected, S-locally-primitive graph with S a nonabelian simple group. Set $G = \mathrm{Aut}(S)\cap \mathrm{Aut}\Gamma$ and $C = \mathrm{C}_{\mathrm{Aut}\Gamma}(S)$. The graph Γ and G are said to satisfy Condition $\mathcal{P}$ if $C \neq 1$ and $\mathrm{Aut}\Gamma$ has no quasiprimitive subgroup of type AS containing $C.G$ as its maximal subgroup.*

Remarks on Condition $\mathcal{P}$

(a) Using [7, Theorem 1.2] we shall prove that, for a connected S-locally-primitive graph Γ with S a nonabelian simple group with Γ and G satisfying Condition $\mathcal{P}$, then any quasiprimitive overgroup Y of G in $\mathrm{Aut}\Gamma$ must be of type AS (see Theorem 1.1). From this we conclude that Condition $\mathcal{P}$ does guarantee that either Γ is non-quasiprimitive or $\mathrm{Aut}\Gamma$ has a non-quasiprimitive subgroup Y such that $C.G$ is maximal in Y.

(b) For an arbitrary group C, it is in general quite difficult to verify that Γ and G satisfy Condition $\mathcal{P}$. However, if C is a nilpotent group and if p is a prime divisor of $|C|$, then $C.G$ is a maximal p-local subgroup of Y whenever there exists such a quasiprimitive group Y of type AS in $\mathrm{Aut}\Gamma$. Note that all maximal p-local subgroups of almost simple groups have been determined (see, for example, [1, 5, 13, 16]). So we can find out which Γ and G satisfy Condition $\mathcal{P}$ using a case-by-case check.

The main results of this paper are the following.

Theorem 1.1 *Suppose that* Γ *and* G *satisfy Condition* $\mathcal{P}$*. Then either* Γ *is a non-quasiprimitive graph; or* $\mathrm{Aut}\Gamma$ *is a quasiprimitive group of type* AS *and* $\mathrm{Aut}\Gamma$ *contains a non-quasiprimitive subgroup* Y *such that* $C.G$ *is maximal in* Y*. Further, for any intransitive minimal normal subgroup* N *of* Y*, then:* N *centralizes* S*; or* $N \cap C = 1$ *and* $N = Z_p^n$ *for some prime* p*; and* $S = S(q)$ *is a simple group of Lie type over a field of order* $q = p^e$*, and further,* S *and* n/e *are as in Table* 1*. In particular, if* S *lies in the first column of Table* 1 *or* $S = E_7(q)$*, then* N *is the unique intransitive minimal normal subgroup of* $\mathrm{Aut}\Gamma$ *not centralized by* S*.*

S	values for n/e		S	values for n/e
$B_l(q), l = 3$ or 4	$[8,9]$ or $[8,16]$		$E_7(q)$	$[56,63]$
$C_l(q), l = 3$ or 4	$[8,9]$ or $[9,16]$		$A_l^{\pm}(q),\ l > 1$	$[l+1, l(l+1)/2]$
$D_l^{\pm}(q), l = 4$ or 5	$[8,12]$ or $[16,20]$		$B_l(q),\ l > 4$	$[2l+1, l^2]$
${}^3D_4(q),\ q$ odd	12		$C_l(q),\ l \neq 1,3,4$	$[2l, l^2]$
$E_6^{\pm}(q)$	$[27,36]$		$D_l^{\pm}(q),\ l \neq 1,2,4,5$	$[2l, l(l-1)]$

Table 1
where $[i,j]$ stands $i \leq n/e \leq j$, for distinct integers i and j with $i < j$.

For Γ and G given as in Theorem 1.1, let Γ^* denote the quotient graph modulo the C-orbits on $V\Gamma$ (obtained by taking C-orbits as vertices and joining two C-orbits by an edge if there is at least one edge in Γ joining a point in the first C-orbit to a point in the second one). By [17], Γ is a cover of Γ^*. As an application of Theorem 1.1 we have the following.

Theorem 1.2 *Let* Γ *and* G *satisfy Condition* $\mathcal{P}$*. Suppose that* C *is a cyclic group with prime order and that* $\mathrm{Aut}\Gamma$ *has no a subgroup* $N{:}S$ *given as in Table* 1*. If* $\mathrm{Aut}\Gamma^*$ *is quasiprimitive of type* AS *with socle* S*, then* Γ *is non-quasiprimitive and* $\mathrm{Aut}\Gamma = C.G$*.*

Remark on Theorem 1.2

For each graph given as in [8, 14], we know that $C = Z_3$ or Z_2 and that $\mathrm{Aut}\Gamma$ has no subgroup $N.S$ given as in Table 1. It is also easy to prove that $S \leq \mathrm{Aut}\Gamma^* \leq \mathrm{Aut}(S)$ (up to isomorphism). By Theorem 1.2, to show $\mathrm{Aut}\Gamma = C.G$, it remains only to verify that Γ and G satisfy Condition $\mathcal{P}$, which can be completed by using the method mentioned in Remark (b) on Condition $\mathcal{P}$. Indeed, this is not difficult to check for the graphs given in [8, 14].

Theorems 1.1 and 1.2 are proved in the next section. In the final section we consider question (2) raised at the beginning of this paper and give some examples of G-quasiprimitive graphs such that $\mathrm{Aut}\Gamma$ and G have same quasiprimitive type but $soc(\mathrm{Aut}\Gamma) \neq soc(G)$.

2 Proof of Theorems 1.1 and 1.2

The first lemma is used in the proof of Theorem 1.1. Its proof follows immediately from [17] and the connectivity of Γ.

Lemma 2.1 *Let Γ be a connected, nonbipartite, Y-locally-primitive graph. Then each intransitive normal subgroup of Y is semiregular.*

Suppose now that G is an almost simple group with socle S and let G and Γ be given as in Theorem 1.1. Write $X := \mathrm{Aut}\Gamma$ and $C := \mathrm{C}_X(S)$. Since S is locally primitive on $V\Gamma$, S is transitive on the arcs of Γ, and so S is not regular on $V\Gamma$. It follows that C is not transitive on $V\Gamma$ (see, for example, [9, Proposition 2.4] or [21]).

Proof of Theorem 1.1 First we show that, for each overgroup K of S in X, if K is quasiprimitive then K is of type AS. Suppose to the contrary that K is not of type AS. By [7, Theorem 1.2], either $\Gamma = K_8$ with $S = \mathrm{PSL}(2,7)$ and $K = \mathrm{AGL}(3,2)$; or K is of type PA with $soc(K) = S_1 \times S_2 \cong S \times S$ and S is a diagonal subgroup of $soc(K)$. In the former case, by [6], $S_\alpha = Z_7{:}Z_3$ and S_α is a maximal subgroup, and hence S acts primitively on $V\Gamma$. It follows from [21] that $C = 1$, which contradicts Condition $\mathcal{P}$. In the latter case, since $S < soc(K)$, Γ is also $soc(K)$-locally-primitive. Since $S_i \lhd soc(K)$ and $S_i \cong S$, for $i = 1, 2$, the both S_1 and S_2 are transitive by Lemma 2.1. It follows that S_1 and S_2 are regular, which is impossible since $|S_1| = |S| > |V\Gamma|$. So K is of type AS. In particular, take $K = X$, then X is of type AS.

Suppose now that X is quasiprimitive. Since X is of type AS, by Condition $\mathcal{P}$, $G.C$ is not maximal in X, and hence there is a proper subgroup Y of X containing $G.C$ such that $G.C$ is maximal in Y. If Y is quasiprimitive then Y is of type AS, which contradicts Condition $\mathcal{P}$. Thus Y is not quasiprimitive.

Let N be an intransitive minimal normal subgroup of Y with $N \not\leq C$. Since Γ is also a Y-locally-primitive graph, by Lemma 2.1, N is semiregular on $V\Gamma$. If $N < C.G$ then CN/C is isomorphic to a normal subgroup of G, which implies that either $N \leq C$ or S isomorphic to a subgroup of CN/C. The former case contradicts $N \not\leq C$. While in the latter case we conclude that $|N|$ is divisible by $|S|$, which contradicts the fact that N is semiregular. Thus $N \not\leq C.G$ and so $Y = \langle N, C.G\rangle$ (since $C.G$ is maximal in Y). Then arguing as in the proof of [7, Theorem 1.1] we conclude that N is an elementary abelian p-group, for some prime p. If $N \cap C \neq 1$, then $N \cap C$ is normalized by N and $C.G$, respectively. Thus $N \cap C$ is a nontrivial normal subgroup of Y. Since $N \cap C \leq C$ and N is a minimal normal subgroup of Y, $N \cap C = N$ and hence $N \leq C$, which is not the case. So $N \cap C = 1$. It follows that S acts faithfully by conjugation on N. Thus S has a faithful projective p-modular representation of degree n. A similar argument as in the proof of [10, Theorem 1.1] shows that S and n/e are given as in Table 1. For example, we look at the case where $S = A_l(q)$ with $q = p_1^e$ for some prime p_1. If $p_1 \neq p$ then $n/e \geq (q-1)/(2, q-1)$ by [13, Table 5.3.A], and hence

$$|N| \geq q^{(q-1)/(2,q-1)} \geq |A_l(q)| > |V\Gamma|,$$

which is impossible. So $p_1 = p$. Now let $R_p(S)$ denote the minimal dimension of a faithful, irreducible, projective KS-module, where K is an algebraically closed field of characteristic p. By [13, Table 5.4.C], $R_p(A_l(q)) = l+1$, which implies that $n/e \geq l+1$. On the other hand, since a Sylow p-subgroup of $A_l(q)$ has order $q^{l(l+1)/2}$ and since $|S|$ is divisible by $|N|$, $n/e \leq l(l+1)/2$. So n/e lies in $[l+1, l(l+1)/2]$. A similar argument deals with other nonabelian simple groups S.

Finally, for S in the first column of Table 1 or $E_7(q)$, if there is another intransitive minimal normal subgroup K not centralized by S, then a similar argument as above implies that $K \cong Z_p^m$ with m/e in the second column of Table 1 or $[56, 63]$. Set $W = NK$. Now W is a normal p subgroup of X with $|W| = p^{n+m}$. Since p^{n+m} is greater than the p-part of $|S|$, W must be transitive on $V\Gamma$ by Lemma 2.1. Recall that S is one of the following groups: $B_l(q)$ or $C_l(q)$ with $l \in \{3,4\}$, $D_l^{\pm}(q)$ with $l = 4$ or 5, ${}^3D_4(q)$ with q odd or $E_7(q)$. From [11, Corollary 2] it follows that $S = \mathrm{PSp}(4,3)$ and $|V\Gamma| = 27$, and so $X \leq \mathrm{AGL}(3,3)$. However it is trivial to see that $\mathrm{PSp}(4,3)$ is not isomorphic to a subgroup of $\mathrm{AGL}(3,3)$, which is a contradiction. So N is the unique intransitive minimal normal subgroup of X not centralized by S. $\square$

Proof of Theorem 1.2 We claim first that $\mathrm{Aut}\Gamma$ is not quasiprimitive. If this is not the case, by Theorem 1.1, $\mathrm{Aut}\Gamma$ contains a non-quasiprimitive subgroup Y such that $C.G$ is maximal in Y. Let N be an intransitive minimal normal subgroup Y. If $N \not\leq C$, by Theorem 1.1, we conclude that $N{:}S$ would be in Table 1, which is not the case. So $N \leq C$ and hence $N = C = Z_{p_1}$. Since $G \cong (C.G)/C < Y/C$ and Y/C is isomorphic to a subgroup of $\mathrm{Aut}\Gamma^*$, $S \leq Y/C \leq \mathrm{Aut}(S)$ (up to isomorphism). It follows that $Y \cong C.G_1$, for some group G_1 with $S \leq G_1 \leq \mathrm{Aut}(S)$. On the other hand, from the definition of G we know that G is the maximal group among the groups H with the properties that $S \leq H \leq \mathrm{Aut}(S)$ and $C.H \leq \mathrm{Aut}\Gamma$, which implies that $G_1 \cong G$. Thus $Y = C.G$, which is a contradiction. So $\mathrm{Aut}\Gamma$ is non-quasiprimitive.

Let N be an intransitive minimal normal subgroup of $\mathrm{Aut}\Gamma$. If $N \not\leq C$ then S acts nontrivially by conjugation on N. Now N is semiregular on $V\Gamma$ by Lemma 2.1. Then arguing as in the proof of Theorem 1.1, we conclude that $N = Z_p^n$, for some prime p and integer $n > 1$, and $N{:}S$ lies in Table 1, which is a contradiction. So $N \leq C$, and hence $N = C$. Now $C \lhd \mathrm{Aut}\Gamma$ and $\mathrm{Aut}\Gamma/C$ is a subgroup of $\mathrm{Aut}\Gamma^*$. Thus $\mathrm{Aut}\Gamma \cong C.G_1^*$, for some subgroup G_1^* of $\mathrm{Aut}\Gamma^*$ with $S \leq G_1^* \leq \mathrm{Aut}(S)$ (up to isomorphism). From the definition of G it follows that $G_1^* \cong G$ and hence $\mathrm{Aut}\Gamma = C.G$. $\square$

3 Some examples of quasiprimitive graphs

In this section we always assume that G is an almost simple group and Γ is a connected G-arc transitive graph with valency d_Γ. By [20] there exists a 2-element $g \in G$ with the properties: $g \notin \mathrm{N}_G(H)$, $g^2 \in H$ and $\langle H, g\rangle = G$, such that

$\Gamma \cong \Gamma^* := \Gamma(G, H, HgH)$ with $d_\Gamma = |H : H \cap H^g|$, where Γ^* is defined by

$$V\Gamma^* = \{Hx \mid x \in G\}, \quad E\Gamma^* = \{\{Hx, Hy\} \mid x, y \in G, xy^{-1} \in HgH\}. \qquad (1)$$

All connected regular G-arc transitive graphs considered in this section will be defined in terms of a subgroup H and a 2-element g as in (1).

Now we give some examples of G-quasiprimitive graphs. The first example gives all G-quasiprimitive graphs with a prime power number of vertices, for G a nonabelian simple group. The proof of Example 3.1 follows immediately from [11, Corollary 2] and [6].

Example 3.1 *Let Γ be a finite connected graph and suppose that G is a transitive subgroup of* $\mathrm{Aut}\Gamma$ *with G a nonabelian simple group. If $|V\Gamma|$ is a prime power, then either Γ is a complete graph or $G = \mathrm{PSU}(4,2)$ and $|V\Gamma| = 27$. Further, G and $soc(\mathrm{Aut}\Gamma)$ are given in Table 2.*

G	$\lvert V\Gamma\rvert$	$soc(\mathrm{Aut}\Gamma)$	comments on G_α
A_{p^a}	p^a	A_{p^a}	$G_\alpha \cong A_{p^a-1}$
$\mathrm{PSL}(n,q)$	$\frac{q^n-1}{q-1} = p^a$	A_{p^a}	the stabilizer of a line or hyperplane
$\mathrm{PSL}(2,11)$	11	A_{11}	$G_\alpha \cong A_5$
M_{23}	23	A_{23}	$G_\alpha \cong M_{22}$
M_{11}	11	A_{11}	$G_\alpha \cong M_{10}$
$\mathrm{PSU}(4,2)$	27	$\mathrm{PSU}(4,2)$	$G_\alpha \cong 2^4{:}A_5$

Table 2

The graphs on the first five lines of Table 2 are all complete, but the graph for $\mathrm{PSU}(4,2)$ is not. The graphs on lines two to five provide one infinite family and three particular graphs with the property that $soc(G) \neq soc(\mathrm{Aut}\Gamma)$.

Example 3.2 *Let Γ be a connected G-arc transitive graph of valency d_Γ, where $(G, |V\Gamma|)$ is one of $(\mathrm{P\Gamma L}(2,8), 36)$, $(A_9, 120)$ and $(M_{11}, 55)$. Then* $\mathrm{Aut}\Gamma$ *is an almost simple group. Moreover, the pair $(G, soc(\mathrm{Aut}G))$ is $(\mathrm{PSL}_2(8), A_9)$, $(A_9, \Omega_8^+(2))$ or (M_{11}, A_{11}), respectively.*

Proof. For $\alpha \in V\Gamma$ write $H = G_\alpha$. For $(G, |V\Gamma|)$ given as above, by [6] we have the structure of H as given in Table 3 (see Column 4). Note that H is a maximal subgroup of G, for G and G_α given as in Table 3. So $\langle H, g\rangle = G$, for any 2-element $g \in G\backslash H$, and hence the graph $\Gamma = \Gamma(G, H, HgH)$ is a connected G-arc transitive graph with valency $d_\Gamma = |H : H \cap H^g|$. Look first at $G = \mathrm{P\Gamma L}(2,8)$ and $H = R{:}T$ with $R \cong Z_7$ and $T \cong Z_6$. Computation shows that T is self-normalized in G. Thus $|H \cap H^g|$ is either 2 or 3, and so $d_\Gamma = 21$ or 14. Moreover, since a Sylow 2-subgroup of G is elementary abelian all suitable 2-elements g have order 2. If $d_\Gamma = 21$ then $H \cap H^g = 2$. Using the Magma program (see Figure 1) we obtain both all connected $\mathrm{P\Gamma L}(2,8)$-arc transitive graphs with valency 21 and their

```
H := PGammaL(2,8);
A :=SylowSubgroup(H, 7);
M :=Normalizer(H, A);
for x in M do
  if Order(x) eq 3 then
     for y in M do
       if Order(y) eq 2 then
       M6:=sub<H | x,y>;
         if Order(M6) eq 6 then
         B:=M6; break;
         end if;
       end if;
     end for;
   end if;
end for;

B3:=SylowSubgroup(B,3);
B2:=SylowSubgroup(B,2);
A21:=sub<H |A, B3>;
N2:=Normalizer(H,B2);

phi, G := CosetAction(H, M);
T := Stabilizer(G, 1);
A21 := phi(A21);
B2 := phi(B2);
NB2 := Normalizer(G, B2);

Grphs := [];
for g in NB2 do
   if Order(g) eq 2 and sub< G | T, g > eq G then
      print "Success with", g;
      found := true;
      Nbs := { 1^(g*x) : x in A21 };
      Gr := Graph< Support(G) | <1, Nbs>^G >;
      print Gr;
      print (AutomorphismGroup(Gr));
      print Order(AutomorphismGroup(Gr));

      Append(~Grphs, Gr);
   end if;
end for;
```

Figure 1: MAGMA program for $\mathrm{P\Gamma L}(2,8)$-arc transitive graphs of valency 21

full automorphism groups. Using a similar method we obtain Table 3, and the conclusion follows from Columns 4 and 5 of Table 3. □

Remark on Example 3.2 Recall the definition of 2-closure of a finite permutation group: if G is a finite permutation group on a set Ω the 2-closure $G^{(2)}$ of G is the largest subgroup of $\mathrm{Sym}(\Omega)$ containing G which has the same orbits as G in the induced action on $\Omega \times \Omega$. For each group G in Example 3.2, Table 3 shows that $Aut\Gamma \cong G^{(2)}$, and all pairs of $(G, \mathrm{Aut}\Gamma)$ in the example occur in [15, Table 1].

G	d_Γ	$\lvert V\Gamma\rvert$	G_α	$soc(G)$	$\mathrm{Aut}\Gamma$
$\mathrm{P\Gamma L}(2,8)$	14 or 21	36	7:6	$\mathrm{PSL}(2,8)$	S_9
A_9	56 or 63	120	$\mathrm{PSL}(2,8).2$	A_9	$\Omega_8^+(2)$
M_{11}	18 or 36	55	$3^2{:}Q_8.2$	M_{11}	S_{11}

Table 3

Acknowledgements

We are grateful to John Cannon for assistance in setting up the Magma programs for calculating general arc-transitive graphs. The second author was partially supported by the Australian Research Council.

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