# Groups in Durham

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My overseas trip in July and August this year had a two-fold purpose. I attended the London Mathematical Society Symposium on Computational Group Theory, held at the University of Durham; and I visited various sites which have Network Systems Corporation HYPERchannel equipment. In this report I deal with the group theoretic part of my travels.

The Symposium was held from Friday, 30 July to Monday, 9 August. There were 40 lectures, presented every morning and afternoon for 8 days, except for a Wednesday rest day. Access to computers was available 24 hours a day every day. I presented two talks: the first, 'Computer implementation of Todd-Coxeter algorithms' on the first Sunday and the second, 'Distinguishing knots', on the second Sunday.

C C Sims of Rutgers and M R Vaughan-Lee of Oxford presented some new results of particular interest to me. Sims described proposals for new types of coset enumeration algorithms and also provided some information about algorithmic behaviour. In particular, he showed that coset enumeration is worse than polynomial. Vaughan-Lee described ways of reducing the number of equations which need to be checked in implementations of the nilpotent quotient algorithm. As far as consistency equations are concerned, he reduced the number of necessary equations from $O(n^3)$ to $O(n^2)$, and $O(n^2)$ is easily shown to be the best possible.

There was excellent computer availability during the conference. The main machine used by participants was an IBM 370/168 at Newcastle. This machine provides the Northumbrian Universities Multiple Access Computing (NUMAC) service running the Michigan Terminal System (MTS) operating system. There was also access via dial-up lines to various networks including the UK Package Switched Service (PSS) with links to machines all over the kingdom.

I took with me the latest versions of coset enumeration and nilpotent quotient algorithm programs developed in Canberra. These programs (and others brought by other people, including the group theory language CAYLEY, the character algorithm system CAS, and the soluble group operating system SOGOS) were implemented on various machines, and some points about program portability were re-emphasized.

Fortran 66 and supersets remain the most portable languages for computational group theory programs. People using other languages had all sorts of problems. In particular, Fortran 77 was not available at NUMAC.

The two programs (both written in a Fortran 66 superset) that I took with me ran under MTS with a couple of minor problems. First, the nilpotent quotient algorithm program failed with an array bounds error. I tracked this down to an optimizer problem. The compiler used initially was a Fortran level H version with optimization turned on by default. The optimizer pulled something out of a loop inappropriately which caused the error. Recompilation with optimization off resolved the problem. Second, both the Todd-Coxeter and nilpotent quotient algorithm programs can write out various information to checkpoint files, which may be formatted or unformatted. Use of formatted checkpoint files proved uncomfortable because of an MTS 'feature'. Under MTS, formatted output fields which correspond to undefined variables are filled with U's. On subsequent input these U's cause error conditions (interactively correctible). Both programs did in fact write out some irrelevant undefined variables to the checkpoint file and then endeavour to restore these, causing trouble under MTS.

Other people struck some problems which are instructive. Do you think that the Fortran function IABS always returns a non-negative value? On the NUMAC IBM 370/168 (and also on the CSIRO Facom M-180N) you can generate the integer −2147483648. If you apply IABS to this, the result is still −2147483648, a consequence of two's complement arithmetic and poor exception reporting. The Fortran user on these machines is presented with no error indication, which is the bad news. One of the conference participants was humbled for some time by an undeclared variable whose default typing was inappropriate. The cost of Fortran's default typing policy has been untold hours of debugging. Of course had the program involved conformed to the draft *DCR Standards Manual* (section 3.4.1, strong typing of variables) this problem would have been revealed by the compiler.

I implemented the Vaughan-Lee consistency check reduction on the spot in the nilpotent quotient algorithm program on the NUMAC machine. Since his consistency equations are a subset of those previously used, it follows that this was necessarily beneficial; and a worthwile reduction in execution time was observed for the test deck. The reduced set of consistency equations will be specially beneficial for large problems. Vaughan-Lee also suggested different, shorter sets of relations to be utilized in exponent checking. He reports time savings with these also, but I have not yet implemented them myself.

At the Symposium, N L Biggs of Royal Holloway College presented a talk entitled 'Presentations for cubic graphs'. Biggs raised some unanswered questions about automorphism groups of cubic graphs. In particular he asked about two groups designated by $5^+(a^{16})$ and $4^+(a^{12})$. He showed that the order of $5^+(a^{16})$ was a multiple of $2^{16}.30.48$ and that the order of $4^+(a^{12})$ was a multiple of $2^8.14.24$, and asked for more information.

It was clear to me that these groups could be usefully investigated with the computer programs that I had taken with me together with some that I had left behind, namely Reidmeister-Schreier and Tietze transformation programs. Fortunately I had left copies of these programs at various places in the United Kingdom on a previous visit in 1978. I was delighted to find these available, running and improved in the care of E F Robertson at the University of St Andrews.

Biggs talked at 3:20 pm on Tuesday 2 August. That night Robertson and I attacked $5^+(a^{16})$ on a St Andrews VAX via a dial-up line, with a little help from CAYLEY and the NUMAC machine. We showed that $5^+(a^{16})$ is practically infinite. We continued the next afternoon and evening attacking $4^+(a^{12})$, using the St Andrews VAX via PSS and also via dial-up lines, and showed that it too is practically infinite. Specifically we showed that $5^+(a^{16})$ has order which is a multiple of $90.16.2^{138}$ and that $4^+(a^{12})$ has order which is a multiple of $14.2.3.16.2^{73}$.

As a result of the Symposium, I intend to submit three papers for publication in the proceedings. They are: 'Two groups which act on cubic graphs' (with E F Robertson), 'Distinguishing eleven crossing knots (with L G Kovacs) and 'A Tietze transformation program' (with P E Kenne, J S Richardson and E F Robertson).

On the lighter side, an outing was arranged for Symposium participants on the Wednesday to Hadrian's Wall and other historic sites in Northumbria. This was enjoyed by all, except perhaps for the absent-minded mathematician who slipped from the wall and sprained an ankle. By the way, do you know the etymology of the word 'Symposium'? The Shorter Oxford English Dictionary says, in part, that the word means 'convivial meeting for drinking, conversation and intellectual entertainment', and the participants endeavoured to live up to the definition fully, with warm British ale available into the late evening.

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A gaggle of group theorists at Durham.