Accurate Self-Localization in Transit Stations: A Case Study

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***Abstract*—Public transit stations and hubs can be difficult to navigate by people who are blind or have low vision, and by people with cognitive impairment. We are building a system, named RouteMe2, that provides microrouting and guidance in these environments. A critical component of this system is self-localization. In this paper, we present a system for self-localization in outdoor places (such as a train station), in which GPS signal is available but, due to shading from nearby buildings, often unreliable. We propose a new approach that uses a small number of Bluetooth low energy (BLE) beacons to increase self-localization accuracy by means of statistical fusion with data from GPS, paired with a Bayes discrete filter tracker. A number of experiments were conducted at San Jose Diridon light rail station to quantitatively assess the performance of the proposed system.**

I. Introduction

Public transit, when available, represents a safe, economical, and environmentally responsible way to travel. It is also the preferable, and sometimes the only, means of transportation for those who cannot drive, and who cannot rely on family or friends to be driven to places. These include people who are old and lost their driver license due to medical reasons; people who are blind or have severe vision conditions (low acuity, tunnel vision); and people with a certain level of cognitive impairment (e.g. early stages of dementia). Although private transportation, in the form of taxi cabs or ride hailing services such as Uber and Lyft, is often available, at least in urban areas, this may be too expensive for regular use by people with low income. Paratransit is an option for those who have a disability; but it is not an ideal solution, as paratransit service has limited coverage and require reservation long in advance of a trip.

Traveling by public transit, though, may be difficult for some people, and especially for those who need it the most. In many cases, the problem is one of information access. Managing a trip, especially one that requires one or more transfers, requires prior knowledge of where and when to catch each vehicle. While en route, travelers must acquire and process different types of information, such as which platform to stand on while waiting for a train, or whether the bus vehicle that just arrived is the correct one. Travelers must maintain continuous awareness of where they are in the scheduled itinerary; make timely decisions (e.g. when to exit a bus); and devise contingency plans when something goes wrong (such as a missed transfer or a delayed arrival). Much of the information available to a traveler is in visual form, such as signs or displays, and thus inaccessible to those who are blind or have low vision. Negotiating difficult or unexpected situations may be cognitively demanding, and may be challenging for those who are anxious, tend to become easily confused, or have trouble processing information in stressful situations.

We are developing a system, called RouteMe2, that is designed to assist travelers using public transit, by providing individualized access to relevant information. RouteMe2 is embodied in a smartphone app and a cloud server system. The software in the app and in the remote server work in tandem to track the traveler, and to provide trip-specific and location-aware information, in a format that is convenient for the user. For example, blind users may receive directional guidance in the form of synthetic speech, with the app guiding them to specific places (e.g. the location of a bus stop) while leveraging landmarks that may be perceivable without sight (such as a bench that is known to be located at the bus stop). For people with cognitive impairment but with usable sight, the app may provide simple directions at each step of the way, possibly relying on visible landmarks ("stand next to the red pole with a bus sign on top"), and allowing them to read or hear the directions as many times as desired. RouteMe2 uses existing trip planning APIs (such as Google Directions or OpenTripPlanner) to determine a route, and tracks the user through the route, re-routing when necessary.

Unlike existing routing and tracking apps already available (e.g. Google Maps, Apple Maps, Transit, Moovit, Citymapper), RouteMe2 is able to generate *microroutes*, that is, pedestrian routes at a small spatial scale. A typical pedestrian route (e.g. from Google Map) normally specifies paths on roads or pedestrian routes to a transit station or bus stop. Sometimes, when this information is available (from a GTFS or NeTEx file), these routes can even specify the bus slot in a large transit center. But rarely, if ever, do these generated routes include small-scale detailed spatial information. Indeed, for blind travelers, who cannot rely on visual landmarks, routes need to be defined at a much finer scale than for sighted people. This is particularly the case in the open, when there are no readily available features that can be perceived by touch (such as a wall, which can be tracked using a long cane) and that can be used to follow a route. For the same reason, it is critical that users be spatially localized within a microroute with enough accuracy for the system to produce meaningful directions.

This contribution describes work on the localization system that will be part of our RouteMe2 app. This localization module is designed to function in challenging situations, of the type that are often found at transit stations. Specifically, we address the case of an outdoor transit station with poor GPS reception due to tall nearby buildings. As well known, the presence of tall structures may obscure view of one or more satellites (*shading*, also known as urban canyon effect). If fewer than 4 satellites are visible, signal from the remaining satellites can only be received via multipath, generating localization errors. Shading is very common in urban environments, and is a major cause of GPS failure. In order to mitigate the effect of shading, we propose the use of Bluetooth Low Energy (BLE) beacons. BLE beacons are a popular tool for localization in indoor, GPS-denied environments. Location information is obtained from the received signal strength (RSSI) from multiple beacons, using a mapping function that is learned in a *fingerprinting* phase. The use of BLE beacons in the outdoors, for situations with poor GPS reception, has received much less attention by the research community. In fact, whereas BLE beacons have been shown to produce acceptable localization accuracy in places characterized by networks of corridors [1], where the user's path is well constrained, they turn out to be much less effective in open spaces, where the large variance of their transmission power often results in large localization error [2].

In our work, we explore the combination of spatial information from GPS and BLE beacons via statistical fusion. We show that, with a proper modeling of the error distribution of the signals involved (spatial location from GPS, RSSI from beacons), it is possible to achieve substantially lower localization error than when using either modality in isolation. In addition, we implemented a spatial tracker based on discrete Bayes filtering. The tracker updates a posterior probability distribution of the user's location over a grid of traversable locations. We demonstrate our algorithms in a case study with a light rail station (VTA Diridon station). Due to a tall building next to the station, GPS signal is poor in a portion of the traversable area, where it gives a large localization error. BLE beacons placed on light poles on both walkable sides of the tracks enable localization, but with high error variance. We study the effect of fusing the two sources of information, as well as the benefit of adding a spatial tracker. In addition, we consider the effect of the density of beacons on the overall error.

II. Related Work

There is increasing interest in systems that enable self-localization in GPS-denied environments (e.g. indoors). A popular approach is to use the measured power of wireless signal from WiFi or Bluetooth (including BLE) beacons. While in principle it could be possible to use power decay models [3], [4] to estimate the distance to a beacon from the measured RSSI from that beacon, then self-localize via multilateration, in practice this is extremely challenging [2] due to issues such as multipath fading (an effect of signal reflection from nearby surfaces) and variations in time of the signal power. For this reason, it is customary to instead "learn" a mapping from the set of received RSSI signal from one or more beacons, to the user's location. This mapping is learned from measurements taken at multiple, known locations, a process called *fingerprinting* [5]–[8].

Localization using RSSI value from Wi-Fi access points (AP) has been widely studied [5]–[7], [9]. This approach leverages the widespread availability of Wi-Fi APs in public environments. However, one generally has no control on the actual density of placements of APs (meaning that some areas of interest may not be covered), or on other factors such as APs being disconnected or moved after fingerprinting. BLE beacons represent a popular alternative to Wi-FI APs [1], [10]–[13]. BLE beacons are generally inexpensive, and being battery-operated they can be placed where desired without wiring concerns. Our work uses BLE beacons for localization in an environment in which GPS data is available, but with poor reliability. Although similar situations are relatively frequent in urban environments, we are not aware of prior work that leverages both sources of information (GPS and BLE beacons) to improve self-localization in these scenarios.

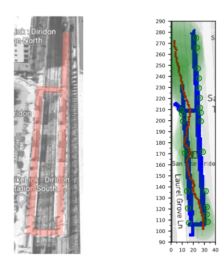

Fig. 1. Left: An aerial picture of the San Jose Diridon light rail station, with the walkable areas highlighted. Right: A GPS track measured while walking on the East platform. Each point is shown with its uncertainty radius (displayed as a green transparent circle). Note the large localization error on the northernmost part of the track.

III. Self-Localization Technique

We define a grid (with square cells 1 meter wide) over the traversable area of the environment being mapped. For the case of the Diridon station considered here, shown in Fig, 1, left, there are two long and parallel pathways (where the East pathway extends North to connect with a pedestrian tunnel under the main station). Travelers can cross the light rail tracks on two "crosswalks", with spring-operated gates. The grid is defined on East, North, Up (ENU) local tangent plane coordinates. A probability distribution of the user's location is defined on this grid using GPS or BLE beacons data as described in the following.

A. GPS Error Modeling

Modern smartphones provide APIs that produce an estimate of the accuracy of GPS data in the form of radius of uncertainty. In first approximation, this could be taken as the standard deviation σ_{GPS} of the GPS localization error. We observed, however, that the uncertainty radius produced by the API is not always reliable. Specifically, there are situations (like the one shown in Fig. 1, right), where the location provided by GPS may have a consistent bias that is poorly modeled by white additive Gaussian noise. For this reason, we decided to use a mixture model instead. Specifically, the probability $p(x_j|GPS)$ of being at a certain location x_j, where x_j is a cell in the grid, is modeled as the convex combination of a normal distribution, centered at the location reported by GPS and with a standard deviation equal to the uncertainty radius, and of a uniform distribution (the latter accounting for large deviations that may occasionally be expected).

B. BLE Beacons RSSI Modeling

Inspired by [14], we model the received power from various beacons at each location x_j in our grid as a normally distributed random vector with independent entries (diagonal covariance). The mean of this random vector is set equal to the average signal vector $\{\overline{RSSI_j^i}\}$ computed from the measurements received within a square region with side of 5 meters (25 cells) centered at x_j (where i indexes the beacons). The entries of the diagonal covariance matrix are set to a constant value σ_B, as this was found to give more stable results than using the empirical variance values. (σ_B was set to 8 dBm in our experiments.) Thus, the conditional likelihood of the received signal can be expressed as:

$$p(\{RSSI^i\}|x_j) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_B} e^{-\frac{(RSSI^i - \overline{RSSI_j^i})^2}{2\sigma_B^2}} \quad (1)$$

Given a measured vector $\{RSSI^i\}$, one can easily compute the posterior distribution over locations under uniform prior: $p(x_j|\{RSSI^i\}) \propto p(\{RSSI^i\}|x_j)$. The most likely location is computed as the cell x_j that maximizes this posterior distribution. Note that this approach involves computing $p(x_j|\{RSSI^i\})$ for all grid cells x_j.

In general, signal from only a limited set of beacons ($\mathcal{B}(x_j)$) will be received at a given location x_j during fingerprinting. (Note that only if we receive at least 3 measurements from the same beacon within the region used to compute the average RSSI, will the beacon be included in $\mathcal{B}(x_j)$.) Let $\mathcal{B}$ be the set of beacons whose signal is received at run time. Care must be taken when computing $p(x_j|\{RSSI^i\})$ for any locations x_j such that the set of beacons with signal received during fingerprinting, $\mathcal{B}(x_j)$, does not match $\mathcal{B}$. In this case, we adopt the following simple strategy: for each beacon in $\mathcal{B}(x_j)$ that is not in $\mathcal{B}$, the missing RSSI value in (1) is set to a small value (-95 dBm). Likewise, for each beacon in $\mathcal{B}$ that is not in $\mathcal{B}(x_j)$, we add a "fake" beacon with the same small value for $\overline{RSSI_j^i}$ in (1).

C. GPS and BLE Beacons Data Fusion

In order to combine localization information from GPS and from the BLE beacons, we employ a standard mechanism of statistical fusion. More precisely, we assume that the signals measured using the two sensing modalities are conditionally independent. Under uniform location prior, this results in a separable posterior distribution:

$$p_{fus}(x_j|\{RSSI^i\}, GPS) \propto p(x_j|\{RSSI^i\})p(x_j|GPS) \quad (2)$$

It is sometimes useful to allocate different "weights" to the two measurements being combined together. We enable this by expressing (2) in the log domain and changing it to a convex combination:

$$\log p_{fus}(x_j|\{RSSI^i\}, GPS) \quad (3)$$
$$= (1-\alpha)\log p(x_j|\{RSSI^i\}) + \alpha \log p(x_j|GPS) + K$$

where $0 \leq \alpha \leq 1$ and K is a normalization constant. Note that smaller values of α assign more weight to the localization estimate from the BLE beacons, and vice-versa. In our experiments, we set $\alpha = 0.2$.

D. Discrete Bayes Tracker

Due to noise and ambiguity (the same RSSI vector may be measured at different locations with similar likelihood), the distribution $p(x_j|\{RSSI^i\})$ (as well as the fused distribution) is often multimodal, with competing peaks that may lead to possibly large "jumps" in the estimated location. In order to overcome this effect, we employed a tracker, which smooths the computed trajectory based on a suitable dynamic prior. (Note that we don't apply the tracker to the location data from GPS, as it is normally already smoothed by the smartphone's API.) Although most recent work in the localization literature uses particle filtering trackers, we opted for a Discrete Bayesian Filter (sometime called Histogram Filter [15]) instead. This is a deterministic algorithm that is appropriate when the spatial domain is discrete (such as a grid) and the dynamic model is also discrete. In our case, we augment the "state" $x(t)$, representing the location of a person at time t, with the person's velocity

$v(t)$. The velocity (more precisely, the displacement within one unit of time, which is assumed to be 1 second) can only take one of a small set of values. More specifically, we assume that from time $t-1$ to time t the user either remains within the same grid cell $x_j(t-1)$, or moves to one of the 8 neighboring cells. With some abuse of notation, we will write $x(t) = x(t-1)+v(t-1)$. The algorithm recursively recomputes the posterior distribution of location and velocity at each time under standard Markovian assumptions as follows:

$$p(x_j(t), v_k(t)|\{RSSI^i(t:)\}) \propto p(\{RSSI^i(t)\}|x_j(t), v_k(t)) \cdot \sum_{\bar{j},\bar{k}} p(x_j(t), v_k(t)|x_{\bar{j}}(t-1), v_{\bar{k}}(t-1)) \; p(x_{\bar{j}}(t-1), v_{\bar{k}}(t-1)|\{RSSI^i(t-1:)\})$$

where k indicates one of the possible 9 values of velocity, and $RSSI^i(t:)$ represents all RSSI readings up to and including time t.

We will assume that the RSSI readings are independent of the user's velocity, and that the user's velocity is independent of his or her location. Under these assumption, the recursion becomes:

$$p(x_j(t), v_k(t)|\{RSSI^i(t:)\}) \propto p(\{RSSI^i(t)\}|x_j(t)) \cdot \sum_{\bar{j},\bar{k}} \delta(x_j(t) - (x_{\bar{j}}(t-1) + v_{\bar{k}}(t-1)) p(v_k(t)|v_{\bar{k}}(t-1)) \; p(x_{\bar{j}}(t-1), v_{\bar{k}}(t-1)|\{RSSI^i(t-1:)\})$$

where $\delta(\cdot)$ is 1 when its argument is 0, 0 otherwise. For what concerns term $p(v_k(t)|v_{\bar{k}}(t-1))$, we will assume that with probability $1-\epsilon$ the velocity remains the same ($k = \bar{k}$), while with probability $\epsilon/8$ it may take any one of the other 8 possible values (where $0 \le \epsilon \le 1$ is a design parameter that was set to 0.2 in our experiments.) Note that, for each location x_j, the recursion only involves a small amount of operations, which are well manageable on a smartphone platform. The algorithm can be easily extended to the case of fused localization from GPS and BLE beacons.

IV. Experiments

A. Setup and Trial Sets

We instrumented the San Jose Diridon light rail station (Fig. 1, left) with 21 BLE beacons (Kontakt Tough Beacon TB15-1) configured as iBeacons and set to the default power level (RSSI of -77 dBm at 1 meter) and advertisement interval of 350 ms. The layout of the beacons can be seen in Fig. 2, left plot. The beacons were placed on light posts, approximately 4 meters tall. Fingerprinting was performed in January of 2019 from RSSI data collected with an iPhone 7. An experimenter walked at constant velocity (approximately 0.5 m/s) over a number of straight paths, while holding the iPhone in her hand. The geodesic coordinates of the endpoint locations of each path were measured using Google Maps, and transformed to grid coordinates. By timestamping the start and end of the walk, the walking velocity was measured, which allowed us to assign a timestamp to each cell in the grid overlapping with the path, and thus to record RSSI measurements for that cell. Overall, data was collected from walking over 22 paths (including walking on the same path in opposite directions). In addition, we mimicked a case with fewer (8) BLE beacons available (see layout of this subset of beacons in Fig. 2, last two plots), by considering data measured only from these beacons.

We tested the performance of our localization system over three different sets of trials. The first set (*Trial Set 1*), collected on the same day of fingerprinting, is formed by 10 trials, where in each trial the experimenter walked at regular speed (approximately 1 m/s) over straight paths, while holding the iPhone in her hand. Timestamped RSSI data was collected during the trials. By measuring the location of the endpoints of these paths, and recording the start and end time of each walk, we were able to estimate the "ground truth" location of the experimenter at all times, and thus to precisely measure the localization error. We report the root mean square distance between the actual location of the experimenter at time t and the estimated location as reported by the system based on the RSSI vector collected at that point. This trial set represents an "ideal" situation, as fingerprinting and data collection were conducted under identical conditions. We tested multiple configurations of the system: GPS tracks, BLE beacons tracks (Sec. III-B), and GPS/BLE beacon fused tracks (Sec. III-C). In addition, for the last two modalities, we experimented with use of the Bayes discrete filter tracker (Sec. III-D). We experimented with all 21 beacons, as well as with the reduced set of 8 beacons.

A second smaller set (*Trial Set 2*) was collected in May of 2019, 4 months after fingerprinting. Of note, all beacons were turned off for a period of time (using the Kontact's beacon management app) between Trial Set 1 and 2, then turned on again before the data collection for Trial Set 2 began. One of the beacons (located on the East platform) stopped working in the process, and thus only 20 beacons were available for Trial Set 2. 4 trials were conducted in an identical fashion as for the previous case, with experimenter walking over straight paths with known endpoints. Path reconstruction was also conducted with the reduced set of beacons (only 7 beacons, due to the aforementioned beacon failure).

The last set (Trial Set 3, collected in July of 2019) is comprised of 3 trials, with the experimenter walking through paths that included multiple turns. For these trials we don't have ground truth measurements of the experiment's location at all times, and thus cannot compute the localization error. However, we recorded the times at which the experimenter took each turn, which allowed us to associate each data point with the segment (between two consecutive turns) the experimenter found herself when that data was collected. Based on this information, we were able to compute all "jumps" – situations in which the system returns a location that is in an incorrect path segment. Our error metric in this case is the proportion of the "jump" events within a path.

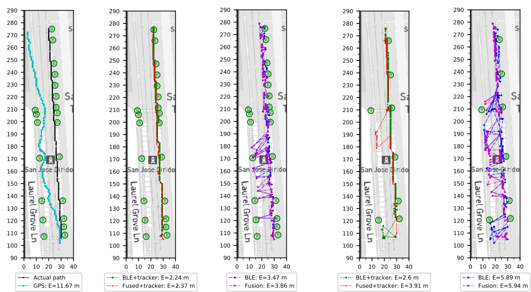

Fig. 2. Experiments with a sample path (East platform) from Trial Set 1. The left plot shows the actual path taken (black) with the track estimated from GPS (light blue) and the average error computed for this path. The next two plots show results using all 21 beacons (whose locations are shown on the map), while the last two plots only 8 beacons are used (locations also shown).

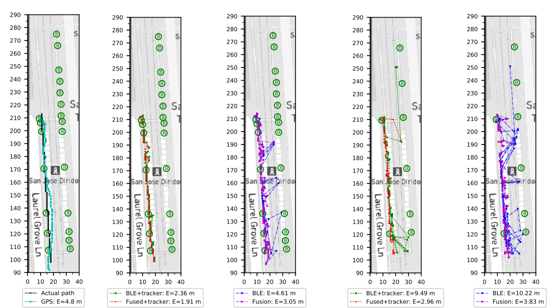

Fig. 3. Experiments with a sample path (West platform) from Trial Set 1. See caption of Fig. 2.

B. Results

Localization errors, averaged over all trials for each trial set, are shown in Tab. I using the chosen metrics. Figs. 2–6 show tracks computed for representative individual paths. A common characteristic of all trial sets is that the GPS tracks were for the most part correct while walking on the West platform, but grossly incorrect when walking on the East platform. This results in the large measured average error reported for the GPS tracks. Localization using only BLE beacons also produces substantial error, especially for the portion of the platforms facing each other. This is due to the fact that, when standing on one platform, the distance to one or more beacons in the other platform is often shorter than the distance to the nearest beacons on the same platform. This generates a multimodal posterior distribution, which results in frequent "jumps" from one platform to the other. In fact, beacon-based localization generates errors also when walking on the West platform, where GPS produces very good results. The situation was aggravated by the fact that, as noted above, one beacon on the East platform stopped functioning after Trial Set 1. In general, when using fewer beacons (see Fig. 5), the error increases, as expected.

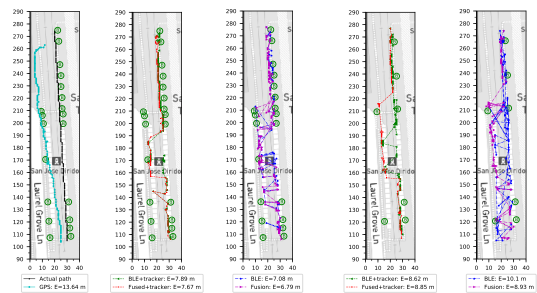

Fig. 4. Experiments with a sample path (East platform) from Trial Set 2. See caption of Fig. 2. Note that, due to a beacon failure, there were only 20 beacons available, of which a subset of 7 beacons was used for the last two plots.

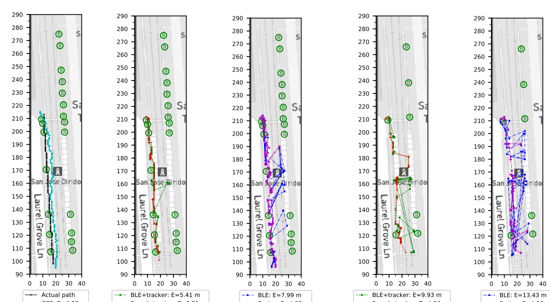

Fig. 5. Experiments with a sample path (West platform) from Trial Set 2. See caption of Fig. 2.

GPS/beacon fusion results in substantially lower overall localization error than with either modality alone, especially when fewer beacons are used. Careful analysis shows that, in this case, fusion mostly contributes to reducing the extent of vertical (North-South) jumps from beacon-based localization. This may be the reason why no apparent benefit is observed from fusion with GPS with respect to using beacons alone in the "jump proportion" metric used for Trial Set 3. Vertical errors within the same segment do not constitute a jump, and are thus not penalized by this metric. In addition, fusion contributes to reducing East-West platform jumps when walking on the West platform.

As expected, the general effect of the tracker is to "stabilize" and smooth the computed paths. Quantitatively, use of the tracker always reduces localization error, although in the case of Trial Set 2, the error reduction is less dramatic than that achieved by GPS/beacon

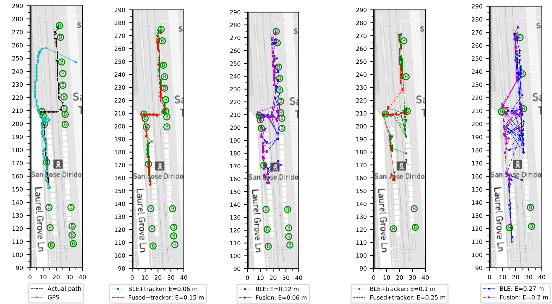

Fig. 6. Experiments with a sample path from Trial Set 3. See caption of Fig. 2.

fusion. Only for Trial Set 1 does use of the tracker on GPS/beacon fused data result in the best results. Some insight on why use of the tracker may not be as effective as expected in Trial Set 2 can be obtained by observation of Fig.4, which shows a case with the experimenter walking on the East platform. In this case, the tracker generates a piecewise smooth trajectory which sometimes places the user on the West platform, amplifying (rather than reducing) the localization error from the beacons. In this particular case, fusion with GPS may even worsen the situation, as seen in the case with 7 beacons.

Full set	GPS	BLE	Fusion	BLE+tracker	Fusion+tracker
Trial Set 1	10.38	4.36	3.83	2.72	2.55
Trial Set 2	7.61	6.65	5.18	6.33	5.81
Trial Set 3	29%	17%	19%	9%	14%

Reduced set	GPS	BLE	Fusion	BLE+tracker	Fusion+tracker
Trial Set 1	10.38	8.10	5.40	5.37	4.24
Trial Set 2	7.61	11.04	6.01	8.72	6.83
Trial Set 3	29%	24%	25%	11%	21%

TABLE I

ERROR COMPUTED OVER ALL TRIALS IN EACH TRIAL SET. THE ERROR IS EXPRESSED AS ROOT MEAN SQUARE DISTANCE BETWEEN ESTIMATED AND ACTUAL LOCATION FOR TRIAL SETS 1 AND 2, AND AS THE "JUMP PROPORTION" FOR TRIAL SET 3. TOP: ALL AVAILABLE BEACONS USED (21 FOR TRIAL SET 1, 20 FOR TRIAL SETS 2 AND 3). BOTTOM: REDUCED SET OF BEACONS USED (8 FOR TRIAL SET 1, 7 FOR TRIAL SETS 2 AND 3).

V. CONCLUSIONS

We have described a system for self-localization in an outdoor location (a light rail station) where GPS signal is available, but often unreliable. This situation is representative of many urban environments, where shading effects reduce the accuracy of GPS-based localization. A set of BLE beacons were installed, with the purpose to enhance self-localization through measurement of the RSSI from these beacons, using a mapping that was learned with a standard fingerprinting phase. Due to the open nature of the place, localization from BLE beacons alone is generally poor, especially when a low density subset of beacons is considered. Statistical fusion of data from GPS and beacons is shown to improve accuracy in most situations, as does use of a Bayes discrete filter tracker with a simple motion model.

In future work, we plan to improve our algorithms to deal with more challenging situations, including: different placements of the smartphone on the user's body (which may lead to masking of the signal from one or more beacons by the user's body); indoor/outdoor transitions; and robustness to sporadic beacons failure (e.g. because of battery depletion).

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