On directional bias for network coverage

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Abstract. Random walks have been proposed as a simple method of efficiently searching, or disseminating information throughout, communication and sensor networks. In nature, animals (such as ants) tend to follow *correlated* random walks, i.e., random walks that are biased towards their current heading. In this paper, we investigate whether or not complementing random walks with directional bias can decrease the expected discovery and coverage times in networks.
To do so, we use a macro-level model of a directionally biased random walk based on Markov chains. By focussing on regular, connected networks, the model allows us to efficiently calculate expected coverage times for different network sizes and biases. Our analysis shows that directional bias can significantly reduce the coverage time, but only when the bias is below a certain value which is dependent on the network size.

1 Introduction

The concept of a random walk was introduced over a century ago by Pearson [11]. Recently, random walks have been proposed for searching, or disseminating information throughout, communications and sensor networks where the network's structure is dynamic, or for other reasons unknown [3, 5, 1, 12]. They are ideal for this purpose as they require no support information like routing tables at nodes [2] — the concept of a random walk being for the agent performing the walk to move randomly to *any* connected node.

The efficiency of random-walk-based algorithms can be measured in terms of the average number of steps the agent requires to cover every node in the network (and hence be guaranteed to find the target node in the case of search algorithms). This is referred to as the *coverage time* under the assumption that the agent takes one step per time unit. Obviously, improving the coverage time for algorithms is an important goal.

For this reason it has been suggested that random walks should be constrained, e.g., to prevent an agent returning to its last visited node, or to direct an agent to parts of the network where relatively few nodes have been visited [7]. We take a similar approach in this paper. We base our movement model on that observed in nature. Many models used by biologists to describe the movement of ants and other animals are based on *correlated* random walks, i.e., random walks which are biased to the animal's current direction [8]. Based on our own observations of ants, we also investigate including a small probability of a non-biased step at any time to model occasional random direction changes.

Directionally biased walks in networks have been investigated by only one other group of researchers. Fink et al. [6] look at the application of directional bias in a cyber-security system in which suspect malicious nodes must be visited by multiple agents. They compare coverage times for directional bias with those for pure random walks, and conclude that directionally biased walks are more efficient. This conclusion, however, is based on micro-level simulation, i.e., direct simulation of agents taking steps, for a single network size and bias. It cannot be generalised to arbitrary network size or bias.

The micro-level simulation approach of Fink et al. requires coverage times to be calculated as the average of multiple runs. They performed 500 simulation runs for each movement model. Such an approach is impractical for a deeper investigation of the effect of directional bias which considers various network sizes and biases. For that reason, in this paper we use a more abstract, macro-level model of a directionally biased walk. It builds on the work of Mian et al. [9] for random walks, describes the directionally biased walk in terms of a Markov chain [10] and allows us to calculate the coverage time for a given network size and bias directly.

2 Directionally biased walks

The investigation in this paper focusses on regular, connected graphs where each node has exactly 8 neighbours. Furthermore, to allow our graphs and hence networks to be finite, we wrap the north and south edges and the east and west edges to form a torus. Our aim is to provide a deeper analysis of directional bias than that by Fink et al. [6] which also investigates the notion on such regular toroidal graphs.

For modelling directional bias in nature, biologists typically use the von Mises distribution [4], a continuous angular function with a parameter κ which affects heading bias. We do not adopt the von Mises distribution in our approach for two reasons. Firstly, we have only a discrete number of directions and so do not require a continuous distribution. Secondly, as in random walks, we would like the computations the agent needs to perform to be simple. Our notion of directional bias limits our agent to choose either its current direction with a probability p (referred to as the *bias*), or any neighbouring direction, i.e., $\pi/4$ radians (45^o) clockwise or anti-clockwise from the current direction, with equal probability of $(1-p)/2$. When the bias p is high, this movement model approximates (discretely) that of the von Mises distribution for a high value of κ.

We also investigate adding occasional random steps to our directionally biased walks. The idea is that with probability r the agent will make a random, rather than directionally biased, step. This better matches our own observations of the movements of ants.

3 A macro-level model

To analyse coverage time under our models of directional bias, we adapt a Markov-chain model [10] and associated formula for coverage time developed for random walks by Mian et al. [9]. Specifically, Main et al. modify the standard Markov-chain model for a random walk so that the starting node is an *absorbing node*, i.e., a node from which the probability of a transition to any neighbour is 0 (and the probability of a transition to itself is 1). They then model the system as starting from the state distribution after the initial distribution, i.e., that in which all neighbours of the starting node have probability $1/n$ where n is the number of neighbours per node. This allows them to calculate the the expected number of nodes covered at any step k directly from the probability of being in the starting node at step k. Coverage time then corresponds to the smallest k where the probability of being in the starting node is 1.

We add an additional dimension to the representation of a network: the current direction of movement. For a network with N nodes, the transition probability matrix for the Markov-chain model is hence no longer of size $N \times N$ but $n * N \times n * N$ where n is the number of neighbours per node (and hence the number of directions of movement). Since there are n positions corresponding to the starting node (one for each direction from which the starting node was entered) there are n absorbing positions in the matrix. Coverage time is calculated in a similar fashion to that of Mian et al. by summing the probabilities of being in these n nodes. Full details of our model can be found in [13].

4 Investigating directional bias

To perform our investigation into directional bias, we plotted graphs of coverage time versus bias (for bias values from 0 to 0.95 in steps of 0.5) for graphs sizes 5×5 (25 nodes) to 15×15 (225 nodes). We calculated the time for coverage of 99% of the network nodes. This was to avoid problems arising with 100% coverage when the coverage converged to a point just below the network size due to inaccuracies in the floating-point arithmetic.

The graph for the 5×5 network is shown in Fig. 1. The horizontal line represents the coverage time for a random walk, and the curved line that for a directionally biased walk under the range of biases. The general shape of the latter and its position in relation to the horizontal line for random bias was consistent for all network sizes in the range considered. A number of interesting results follow from this analysis.

1. The best coverage time is achieved for a bias of 0. This corresponds to an agent which always changes direction by $\pi/4$ radians on every step.
2. While for low directional biases (0 up to around 0.7 for the 5×5 case) coverage time is less than that for a random walk, for higher biases it is greater than that for a random walk.

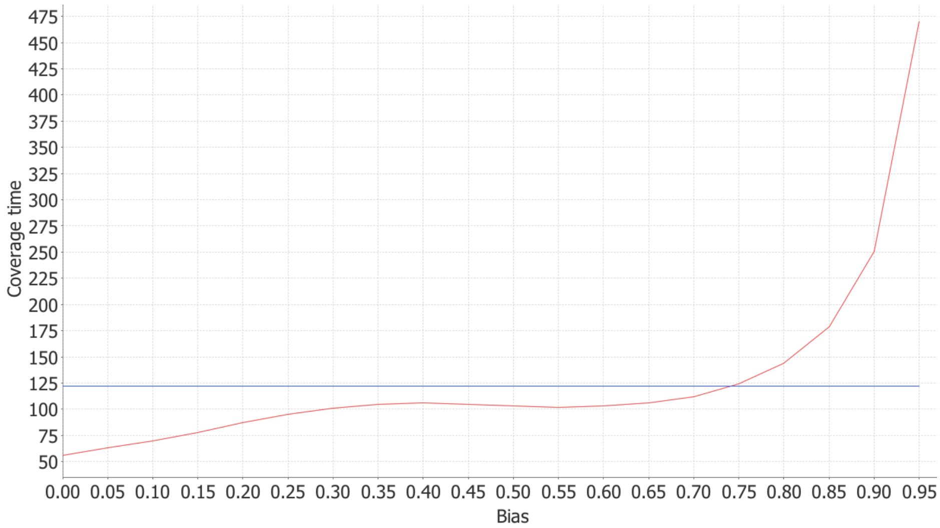

Fig. 1. Random vs directionally biased walk for a network of 5 × 5 nodes.

3. The value of the bias at which a directionally biased walk becomes less efficient than a random walk (from here on called the *cross-over bias*), progressively increases as the size of the network increases. It is around 0.74 for a 5×5 network, and 0.93 for a 15×15 network.
4. The improvement in efficiency of directional bias increases as the size of the network increases. For a directional bias of 0.5 the increase in efficiency is less than 25% for a 5×5 network, and around 60% for a 15×15 network.

Point 1 is particularly interesting as it suggests a new movement model that was not initially anticipated. Our initial motivation was to investigate movement models similar to those observed in nature, which are best represented by a von Mises distribution. However, low values of bias in our movement model (including the value 0) do not approximate a von Mises distribution. The new model, although perhaps impractical as a means of movement in nature, can nevertheless be readily implemented in network search and dissemination algorithms.

Point 2 is also interesting as it indicates that directional bias is only effective in reducing coverage time when the bias is not too large. This result was also unanticipated as directional bias in the movement of animals tends to be high. However, the areas over which such animals move would correspond to networks significantly larger than those we considered. Point 3 anticipates that the cross-over bias would be higher in such networks. This conjecture is supported by the work of Fink et al. [6] whose micro-level simulation of a network of 100 × 100 nodes shows that a directionally biased walk (approximating a von Mises distribution with high κ) is more efficient than a random walk.

The second part of our investigation considered the movement model where occasional random steps are added to a directionally biased walk. The following results emerged from this analysis.

1. As may have been predicted, the addition of random steps moves the coverage time closer to that or a random walk. Hence, for bias values lower than the cross-over bias the coverage time increases, but for values higher than the cross-over value the coverage time decreases. For a 5×5 network and a bias of 0 the coverage time increased from around 55 to 70, and for a bias of 0.95 it decreased significantly from around 470 to about 158.
2. The introduction of random steps increases the cross-over bias. For $r = 0.1$ the cross-over bias increases to 0.78 (from 0.74 for no random steps) and for $r = 0.2$ to 0.84.

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